AN ARBITRARILY BENT MONOPOLE ANTENNA ABOVE A PERFECT INFINITE GROUND PLANE

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THESIS

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ABSTRACT

Hallén type integral equations are formulated for an arbitrarily bent monopole above an infinite perfect ground plane and, by numerical solution techniques, current distributions are obtained. The coupled integral equations are founded upon the familiar electric scalar potential and magnetic vector potential. Numerical results are presented for several cases including different bent angles and are compared where possible with available data.

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I. INTRODUCTION

A linear antenna is one in which a "long dimension" can be discerned; for example, a long circular cylinder the length of which is much greater than the diameter is looked upon as a linear antenna. Initially, the emphasis will be limited to the so-called cylindrical antenna driven at its mid-point by a voltage source V applied to a vanishingly short gap at the antenna center. The antenna will be considered to reside in a lossless, homogeneous and isotropic medium of infinite extent.

A thin wire antenna is commonly defined as a linear antenna which has a radius much less than a wavelength. There are several approximations which may be employed in the analysis of thin wire antennas.

(1) The current on the cylindrical surface is assumed to be entirely in the axial direction.

(2) The current distribution is assumed to be constant around the periphery of the wire at a given cross-section and hence can be approximated as a filament of current on the axis of the cylinder.

(3) The boundary conditions are applied only to those components of the total electric field which are tangent to the wire surface and parallel to the wire axis.

Many investigators have considered the theoretical problem of determining the current induced on a thin wire scattering antenna. [Ref. 1,2,3]. These investigations have been extended to include the treatment of arbitrary configurations of thin wire scattering antennas intersecting at a single junction. [Ref. 4]. Three particular configurations for which numerical results have been obtained are a set of nonintersecting thin

straight wires, [Ref. 5], a set of perpendicular intersecting thin straight wires, [Ref. 6], and a pair of skewed crossed wires. [Ref. 7].

One general approach to finding the current on thin wire antennas is to obtain a set of integral equations which involve the unknown current distributions. Using the equation of continuity, one may determine expressions for the vector and scalar potentials in terms of the current distribution. It is in obtaining these integral equations that a choice can be made.

Following the Pockington equation formulation, the expressions for the vector and scalar potentials are used along with Maxwell's equations to derive a system of integro-differential equations for the unknown current distribution. An auxiliary scalar function is defined and introduced into the system of integro-differential equations. The solution of these now ordinary differential equations subject to appropriate boundary conditions gives the desired set of integral equations for the unknown current distribution.

If the Hallén integral equation approach is used, one independently establishes two expressions for a given component of the vector potential in the direction of a wire element and these expressions are evaluated on the wire surface. One expression is obtained from the solution of an inhomogeneous differential equation satisfied by the component of vector potential.

Another expression for vector potential is obtained directly from the familiar vector potential itself. These two expressions must be equal and, hence, are equated. Next, the boundary condition that the tangential component of electric field on the surface of the wire element must be zero is applied. Now the arbitrary constants of integration which arise

in the solution of the inhomogeneous differential equation are evaluated. The constants of integration must have assumed values which are in compliance with all other required conditions. [Ref. 8].

There have been numerous methods proposed for solving the integral equations, however obtained. These methods have fallen into a few broad catagories:

(1) Iteration [Ref. 9, 10].

(2) Expansion of the current in a Fourier series [Ref. 11].

(3) The variational method [Ref. 12,13,14].

(4) The Wiener-Hopf techniques for the case of the long antenna [Ref. 15].

(5) Numerical methods [Ref 1, 16].

One effective numerical method of solving the integral equation is the so-called direct integration technique (Method of Moments). [Ref. 16, 17]. Basically the technique utilizes an expansion of the unknown function in some convenient manner. This expression may be a finite sum of piecewise continuous functions which are non-zero over some small interval of definition and vanish elsewhere. A substitution of this expression for the unknown function into the system of integral equations reduces them to a system of linear algebraic equations for the unknown coefficients. These linear algebraic equations may be solved by conventional techniques.

The purpose of this thesis is to formulate the coupled integral equations for an arbitrarily bent monopole, above an infinite, perfectly conducting ground plane, in a Hallén-type integral which is founded entirely upon the familiar scalar potential and magnetic vector potential. These integral equations are then solved using the direct integration

technique. Throughout the thesis it is found that the coupled integral equations obtained by the Hallén approach can be readily obtained without employing an auxiliary scalar potential. How the various arbitrary constants, which arise in the development, are related to the electric scalar potential is clearly shown.

II. DERIVATION OF MATRIX EQUATION

In this section the system of integral equations for the current distribution on an arbitrarily bent monopole, above an infinite, perfectly conducting ground plane, driven at its base by a delta gap generator is derived. From the equations, an operator is defined such that the method of moments can be applied to find an approximate representation of the operator by a matrix. The solution for the desired current distribution is obtained by inverting the matrix.

A. FORMULATION OF THE PROBLEM

Figure 1 defines the coordinate system. The antenna consists of two thin wire segments with a single junction. The segments have radii a_z and a_s and lengths h_z and h_s. The primary source is the field in the gap which is maintained at V_o/Δ volts per meter by the generator. The induced source is the current on the dipole. The solution is carried out assuming the wires are perfect conductors. With this assumption the tangential component of the total electric field at the surface of each wire segment must vanish. Due to axial symmetry of each wire segment, the phi-component of each electric field must vanish. Hence for each wire segment the following is obtained:

$$E_z^i(\bar{r}_s) + E_z^p(\bar{r}_s) = 0 \tag{1}$$

$$E_s^i(\bar{\rho}_s) + E_s^p(\bar{\rho}_s) = 0 \tag{2}$$

where $E_z^i(\bar{r}_s)$ is the tangential component of the induced electric field at the surface of the z-directed wire and $E_z^p(\bar{r}_s)$ is the tangential

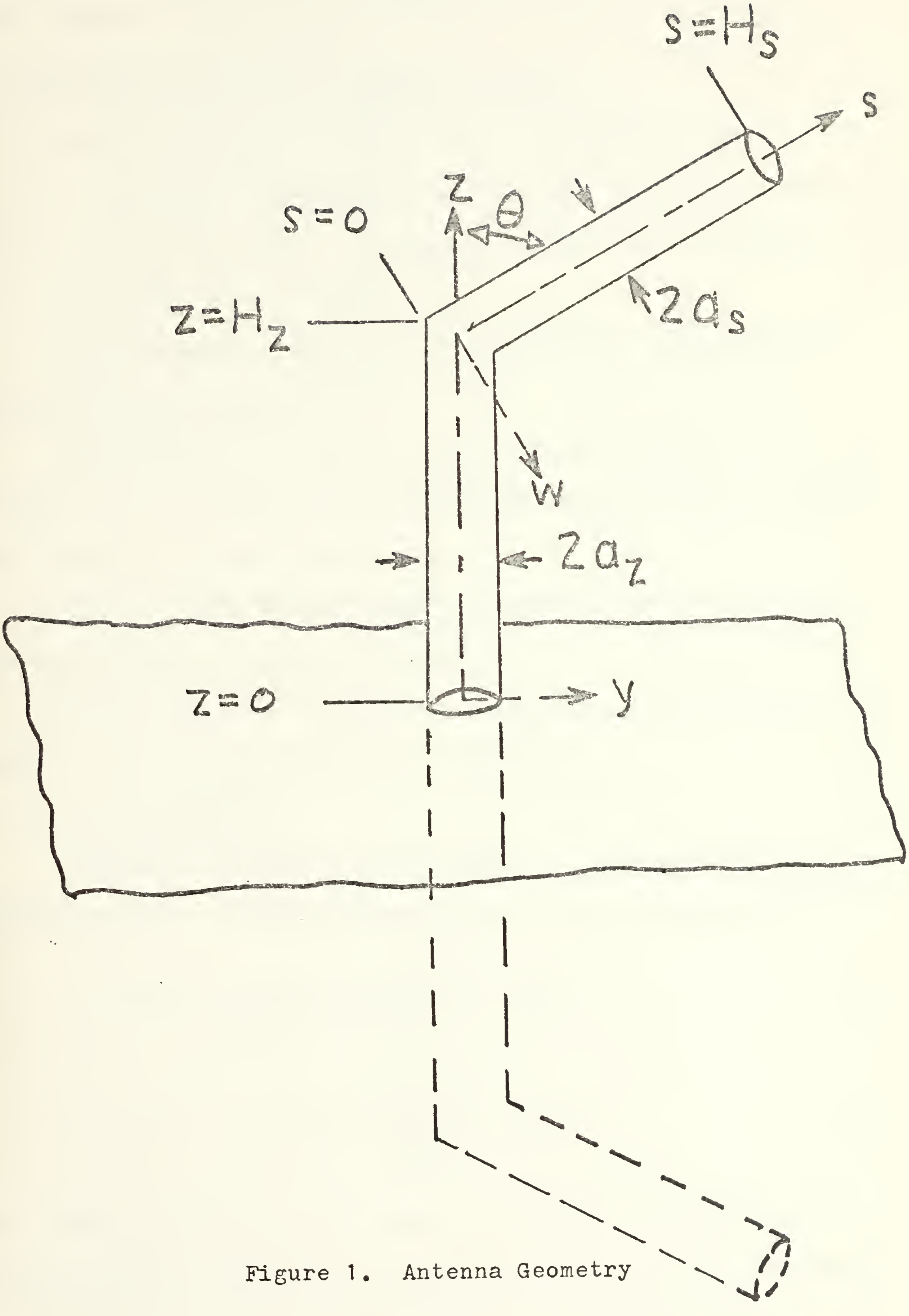


Figure 1. Antenna Geometry

component of the primary electric field at the surface of the z-directed wire. Equation (2) is similarly obtained for the s-directed wire.

B. DERIVATION OF THE COUPLED INTEGRAL EQUATIONS

The Hallén equations are derived from a classical viewpoint for the case of a cylindrical antenna driven at its center by a so-called slice generator. [Ref. 9, 18, 19]. The general procedure for developing a Hallen type equation is as follows:

(1) Determine the magnetic vector potential component from the potential integral equation for $\overline{A}$.

(2) Determine the magnetic vector potential component along the cylinder from the inhomogeneous differential equation which the magnetic vector potential component satisfies.

(3) Write the desired integral equation by equating the two expressions obtained in steps one and two.

(4) Apply appropriate boundary conditions on the surface of the cylinder to the equation obtained in step three.

1. Vector Potentials

Consider the antenna shown in figure 1. Assuming harmonic time dependence ($e^{j\omega t}$), the vector potential due to the antenna current is

$$\overline{A}(\overline{r}) = \frac{\mu}{4\pi} \int_{v'} \overline{J}(\overline{r}') \frac{e^{-jk|\overline{r}-\overline{r}'|}}{|\overline{r}-\overline{r}'|} dv(\overline{r}') \tag{3}$$

$$\overline{A}(\overline{\rho}) = \frac{\mu}{4\pi} \int_{v'} \overline{J}(\overline{p}') \frac{e^{-jk|\overline{\rho}-\overline{\rho}'|}}{|\overline{\rho}-\overline{\rho}'|} dv(\overline{\rho}') \tag{4}$$

Due to the geometry the vector potential has only a z-directed and s-directed component which can be written on the axis of each wire segment

as

$$A_z(z) = \frac{\mu}{4\pi} \int_{z'} I_z(z') \frac{e^{-jkR}}{R} dz' \tag{5}$$

$$A_s(s) = \frac{\mu}{4\pi} \int_{s'} I_s(s') \frac{e^{-jkP}}{P} ds' \tag{6}$$

where $R = |\bar{r}-\bar{r}'|$ and $P = |\bar{\rho}-\bar{\rho}'|$. R and P were evaluated at $\bar{r}'$ and $\bar{\rho}'$ equal to the radius of the respective wire segments with ϕ' equal to zero.

2. Non-Homogeneous Differential Equations

It is now seen that $A_z(z)$ and $A_s(s)$ can not be determined directly from equations (5) and (6) without knowledge of $I(z')$ and $I(s')$. A differential equation is now derived which can be solved for $A_z(z)$ and $A_s(s)$

$$\bar{E} = -j\omega\bar{A} + \frac{1}{j\omega\mu\epsilon} \text{ grad } (\text{div } \bar{A}). \tag{7}$$

The electric fields produced by the currents are, according to equation (7),

$$\bar{E}_z^{\,i}(\bar{r}) = j\omega A_z(\bar{r})\hat{U}_z + \frac{1}{j\omega\mu\epsilon} \text{ grad } \left[\text{div}\left(A_z(\bar{r})\hat{U}_z\right)\right] \tag{8}$$

$$\bar{E}_s^{\,i}(\bar{\rho}) = -j\omega A_s(\bar{\rho})\hat{U}_s + \frac{1}{j\omega\mu\epsilon} \text{ grad } \left[\text{div}\left(A_s(\bar{\rho})\hat{U}_s\right)\right]. \tag{9}$$

Having applied the gradient operation in circular cylindrical coordinates and with axial symmetry eliminating the phi-component of $\bar{E}_z^{\,i}(\bar{r})$ and $\bar{E}_s^{\,i}(\bar{\rho})$, the fact that the total tangential electric field on the surface of the antenna must vanish is applied. The following results are obtained:

$$\frac{\partial^2}{\partial z^2}\left[A_z(\bar{r}_s)\right] + k^2 A_z(\bar{r}_s) = \frac{jk^2}{\omega} E_z^{\,P}(\bar{r}_s) - \frac{\partial}{\partial z}\left[\frac{\partial}{\partial y} A_y(\bar{r}_s)\right] \tag{10}$$

$$\frac{\partial^2}{\partial s^2}\left[A_s(\bar{\rho}_s)\right] + k^2 A_s(\bar{\rho}_s) = \frac{jk^2}{\omega} E_s^{\,P}(\bar{\rho}_s) - \frac{\partial}{\partial s}\left[\frac{\partial}{\partial w} A_w(\bar{\rho}_s)\right], \tag{11}$$

where $\bar{r}_s$ and $\bar{\rho}_s$ denote any point on the z and s-directed wires respectively. Equations (10) and (11) are two coupled partial differential equations because of the last term on the right in each equation. A Hallén solution of these two coupled equations is needed.

3. Solution of Coupled Differential Equations

Equations (10) and (11) are of the form

$$\frac{d^2 f(x)}{dx^2} + k^2 f(x) = u(x) + \frac{\partial v(x)}{\partial x} . \tag{12}$$

The general solution to equation (12) was contained in Ref. [23]. It was found to be

$$f(x) = A \cos kx + B \sin kx + \frac{1}{k} \int_{x'=q}^{x} \left[U(x') + \frac{\partial V(x')}{\partial x'} \right] \sin k(x-x')dx' . \tag{13}$$

The integral portion of equation (13) is zero at x equal q and its derivative is zero at this position. One integration by parts results in

$$\int_{x'=q'}^{x} \frac{\partial V(x')}{\partial x'} \sin k(x-x')dx' = -V(q) \sin k(x-q)$$

$$+ k \int_{x'=q}^{x} V(x') \cos k(x-x')dx' \tag{14}$$

which, when put into equation (13), gives

$$f(x) = A \cos kx + B \sin kx - \frac{1}{k} V(q) \sin k(x-q)$$

$$+ \frac{1}{k} \int_{x'=q}^{x} U(x') \sin k(x-x')dx' + \int_{x'=q}^{x} V(x') \cos k(x-x')dx' . \tag{15}$$

Since V(q) is fixed, the first three terms are combined and new constants obtained are

$$a = A + \frac{1}{k} V(q) \sin kq, \text{ and} \tag{16a}$$

$$b = B - \frac{1}{k} V(q) \cos kq. \tag{16b}$$

Combining equation (16) with equation (15) gives

$$f(x) = a \cos kx + b \sin kx + \frac{1}{k}\int_{x'=q}^{x} U(x') \sin k(x-x')dx'$$

$$+ \int_{x'=q}^{x} V(x') \cos (x-x')dx'. \tag{17}$$

With equation (17) available the solution to equations (10) and (11) is determined with

$$\frac{\partial}{\partial y} A_y(\bar{r}_s) = V(x); \quad \frac{jk^2}{\omega} E_z{}^p(\bar{r}_s) = U(x) \quad \text{for equation (10)}$$

$$\text{and} \quad \frac{\partial}{\partial w} A_w(\bar{\rho}_s) = V(x); \quad \frac{jk^2}{\omega} E_s{}^p(\bar{\rho}_s) = U(x) \quad \text{for equation (11)}$$

which gives

$$A_z(z) = a_z \cos kz + b_z \sin kz + \frac{jk}{\omega}\int_{n=z_o}^{z} E_z(n) \sin k(z-n)dn$$

$$- \frac{1}{k}\int_{n=z_o}^{z}\left[\frac{\partial}{\partial y} A_y(n)\right] \cos k(z-n)dn \quad \text{and} \tag{18a}$$

$$A_s(s) = a_s \cos ks + b_s \sin ks + \frac{jk}{\omega}\int_{n=s_o}^{s} E_s(n) \sin k(s-n)dn$$

$$- \frac{1}{k}\int_{n=s_o}^{s}\left[\frac{\partial}{\partial w} A_w(n)\right] \cos k(s-n)dn. \tag{18b}$$

The $\frac{\partial}{\partial y}\left[A_y(z)\right]$ means $A_y(\bar{r})$ is differentiated with respect to y and then has to be evaluated along a z-directed line on which it is dependent. This is similarly true for $\frac{\partial}{\partial w}\left[A_w(s)\right]$. The arbitrary constants of equation (18a) are a_z, b_z and z_o and any one of these three may be specified directly, but the other two must comply with physical boundary conditions. This applies to equation (18b) and constants a_s, b_s and s_o also.

4. Scalar Potential

With equations (18a) and (18b) one faces the problem of what to do about the constants of integration. One can enforce the boundary condition that the scalar potential must be continuous at each wire segment junction to eliminate a constant or relate two constants. To obtain such an expression is the next task.

Differentiating equation (18a) with respect to z and (18b) with respect to s according to Leibnitz's rule [Ref. 24] gives

$$\frac{\partial}{\partial z} A_z(z) = -ka_z \sin kz + kb_z \cos kz + \frac{\partial}{\partial z}\left\{\frac{jk}{\omega}\int_{n=z_o}^{z} E_z(n) \sin k(z-n)dn\right\}$$

$$- \frac{\partial}{\partial y} A_y(z) + k\int_{n=z_o}^{z}\left[\frac{\partial}{\partial y} A_y(n)\right] \sin k(z-n)dn \tag{19a}$$

and

$$\frac{\partial}{\partial s} A_s(s) = -ka_s \sin ks + kb_s \cos ks + \frac{\partial}{\partial s}\left\{\frac{jk}{\omega}\int_{n=s_o}^{s} E_s(n) \sin k(s-n)dn\right\}$$

$$- \frac{\partial}{\partial w} A_w(s) + k\int_{n=s_o}^{s}\left[\frac{\partial}{\partial w} A_w(n)\right] \sin k(s-n)dn. \tag{19b}$$

The Lorentz condition relates the magnetic vector potential $\overline{A}$ to the electric scalar potential and yields

$$-\frac{jk^2}{\omega}\phi(\overline{r}) = \frac{\partial}{\partial y} A_y(\overline{r}) + \frac{\partial}{\partial z} A_z(\overline{r}) \quad \text{and} \tag{20a}$$

$$-\frac{jk^2}{\omega}\phi(\overline{\rho}) = \frac{\partial}{\partial w} A_w(\overline{\rho}) + \frac{\partial}{\partial s} A_s(\overline{\rho}) . \tag{20b}$$

Rearrangement of equations (19a) and (19b) gives

$$-\frac{jk^2}{\omega}\phi^z(z) = -ka_z \sin kz + kb_z \cos kz + \frac{\partial}{\partial z}\left\{\frac{jk}{\omega}\int_{n=z_o}^{z} E_z(n) \sin k(z-n)dn\right\}$$

$$+k\int_{n=z_o}^{z}\left[\frac{\partial}{\partial y} A_y(n)\right] \sin k(z-n)dn \tag{21a}$$

and

$$-\frac{jk^2}{\omega}\phi^s(s) = -ka_s \sin ks + kb_s \cos ks + \frac{\partial}{\partial s}\left\{\frac{jk}{\omega}\int_{n=s_o}^{s} E_s(n) \sin k(s-n)dn\right\}$$

$$+k\int_{n=s_o}^{s}\left[\frac{\partial}{\partial w} A_w(n)\right] \sin k(z-n)dn. \qquad (21b)$$

which is valid along the z- and s-directed lines respectively.

$\phi^z(z)$ is the total scalar potential along the z- directed wire and $\phi^s(s)$ is the total scalar potential along the s-directed wire, both of which are due to all charges on the wire segments, but the potentials are expressed in terms of the constants of integration. The partial derivatives expressed in equations (21a) and (21b) must now be evaluated.

Going back to equation (13) it is seen that the integral portion of (13) is zero at x equal q and its derivative is zero at x equal q. Now in the derivation these forms are used

$$\frac{1}{k}\int_{x'=q}^{x} U(x') \sin k(x-x')dx' + \int_{x'=q}^{x} V(x') \cos k(x-x')dx' . \qquad (22)$$

Their relationship to the antenna problem is given below equation (17). Now applying these facts the result is

$$\frac{\partial}{\partial z}\left\{\frac{jk}{\omega}\int_{n=z_o}^{z} E_z(n) \sin k(z-n)dn\right\} = 0 \quad \text{for all } z_o \qquad (23a)$$

$$\frac{\partial}{\partial s}\left\{\frac{jk}{\omega}\int_{n=s_o}^{s} E_s(n) \sin k(s-n)dn\right\} = 0 \quad \text{for all } s_o. \qquad (23b)$$

This result implies that one may choose s_o and z_o as desired, but at the same time doing this in a manner to lessen subsequent calculations. Once s_o and z_o are determined the other two constants of integration in each equation must meet boundary conditions.

C. BOUNDARY CONDITIONS

The application of these necessary boundary conditions to the currents on the thin wires should provide the additional equations to yield a unique solution for the current distributions.

1. At The Junction

As has been stated by Taylor [Ref. 4], there are essentially two conditions which must prevail at all junctions:

(1) Kirchoff's current law must hold.

(2) Continuity of the scalar potential must exist.

The first of these requires that there be no buildup or accumulation of charge at the junction. Stated differently, the total charge entering the junction must be equal to the total charge leaving the junction. The second condition requires that the potential on each of the wires at the junction be the same. There can be no jump or discontinuity in scalar potential at a wire junction.

2. At The Free Ends

According to previous considerations of thin wire theory [Ref. 1, 2,4], the current must vanish at the free ends of all wires in the configuration.

III. SIMPLIFICATIONS ARISING FROM GEOMETRICAL SYMMETRY

Any symmetries in the geometry of the antenna elements are reflected in the structure of the generalized impedance matrix Z_{mn} . Judicious use of these symmetries can result in considerable reduction in both computing time and the number of memory locations required for a solution.

A. IMAGE SYMMETRY

Referring to figure 1 the antenna has image symmetry about the plane $z = 0$ through the midsection of the antenna. The boundary condition at a perfect electric conductor is zero tangential components of electric field. An element of source plus an "image" element of source, radiating in free space, produces zero tangential electric field over the plane through the midsection of the antenna. According to uniqueness concepts, the solution to this problem is also the solution for a current element adjacent to a plane conductor. The necessary orientation and excitation of image elements is summarized in many texts. The main point though is that symmetry must be maintained in the image problem. Once image theory is used and the resulting symmetry is recognized, all that is needed is to find the kernel for each "image" current element and combine it algebraically with its corresponding "non-image" current element kernel which reduces the number of required equations by a factor of two.

B. TOEPLITZ PROPERTY

The Toeplitz property of the Z_{mn} matrix can be used to realize considerable economy of computing time. An element of the matrix depends only on the absolute value of the distance between the source point m

and the field point n if there is no bend involved. Since there are only N distinct distances involved, only N different quantities need be calculated for a matrix containing N x N elements. These N quantities are calculated first and the computer is instructed to arrange them in the proper locations to form the complete matrix.

The antenna has a bend so only certain portions of the matrix can be obtained using this property. It is the image of the bend that prevents the entire matrix from exhibiting this property.

IV. NUMERICAL SOLUTION

One of the most general and most useful techniques for solving problems in antenna theory is the direct integration technique. A detailed discussion of the technique has been presented by Harrington [Ref. 16]. Since this basic technique is employed throughout the remainder of this thesis, a short quantitative discussion using the notation of Harrington is included at this time.

A. METHOD OF MOMENTS

The equation to be solved is of the form

$$L(f) = g \tag{24}$$

where L is a linear operator, g is the tangential component of the applied field on each conductor and f is the unknown current to be determined. Using the terminology of Harrington [Ref. 17] it is evident from the uniqueness theorem that the problem is deterministic. Since L and g are given, it is a problem of analysis.

A unifying concept for the solution of such problems is afforded by the method of moments. In order to apply the method, it is necessary to define:

(1) A set of expansion or basis functions $f_1, f_2, \ldots, f_n$ in the domain of L. These known functions must be linearly independent and chosen such that a reasonable approximation to f can be obtained by a finite series expansion of the form

$$f = \sum_{n=1}^{N} \alpha_n f_n \tag{25}$$

where the α_n's are unknown constants to be determined. If equation (25) is substituted into equation (24), one obtains

$$\sum_{n=1}^{N} \alpha_n L(f_n) = g \quad . \tag{26}$$

(2) A set of weighting or testing functions $W_1, W_2, \ldots, W_m, W_{m+1}, \ldots, W_N$ in the range of L and linearly independent must be chosen. If the same set of functions is used for expansion and testing, the procedure is known as Galerkin's method and is equivalent to the Rayleigh-Ritz variational procedure [Ref. 17, 25].

(3) An inner product suitable for the problem. For most electromagnetic field problems, a suitable inner product is

$$\langle W, f\rangle = \int_{n_1}^{n_2} f(n)W(n)\,dn' \tag{27}$$

where n_1 and n_2 are chosen over the region of interest. The resulting expression is

$$\sum_{n=1}^{N} \alpha_n \langle W_m, L(f_n)\rangle = \langle W_m, g\rangle \quad , \quad m = 1,2,\ldots,N \quad . \tag{28}$$

Inspection of equation (28) shows that it can be written as a matrix equation by defining the following matrices

$$[z_{mn}] = \begin{bmatrix} \langle W_1, Lf_1\rangle & \langle W_1, Lf_2\rangle & \ldots & \langle W_1, Lf_N\rangle \\ \langle W_2, Lf_1\rangle & \langle W_2, Lf_2\rangle & \ldots & \langle W_2, Lf_N\rangle \\ \vdots & \vdots & & \vdots \\ \langle W_N, Lf_1\rangle & \langle W_N, Lf_2\rangle & \ldots & \langle W_N, Lf_N\rangle \end{bmatrix} \tag{29}$$

$$[\alpha_n] = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{bmatrix} , \text{ and } [V_m] = \begin{bmatrix} \langle W_1, g\rangle \\ \langle W_2, g\rangle \\ \vdots \\ \langle W_N, g\rangle \end{bmatrix} \quad . \tag{30}$$

The matrix equation may now be written as

$$[z_{mn}]\ [\alpha_n] = [V_m]\ . \tag{31}$$

By analogy with multiport network theory, $[z_{mn}]$ is called a generalized impedance matrix and $[V_m]$ a generalized voltage matrix. Only if $[z_{mn}]$ is nonsingular does its inverse exist and the desired solution is

$$[\alpha_n] = [z_{mn}]^{-1}\ [V_m]\ . \tag{32}$$

Since the α_n's are then known, the solution for the unknown current f is determined by equation (25).

The expediency of obtaining a solution for f depends not only on L and g, which are usually determined by the physical problem, but also on the functions f_n and W_m. Extreme care should be taken in choosing these two sets of functions. Experience with problems of the general type considered in this thesis suggest that only certain particular choices for the expansion functions and for the testing functions yield equations which are amenable to solution. In particular, when the testing functions are chosen to be Dirac delta functions, the resulting technique is usually referred to as point matching.

B. EXPANSION FUNCTIONS FOR THE UNKNOWN CURRENT

There are several popular choices for the expansion functions which have been considered in the literature. Some of the more common of these are piecewise constant, piecewise linear, piecewise parabolic and piecewise sinusoidal [Refs. 20, 21]. All of the aforementioned fall under the general label of the method of subsections. This method has the antenna contour divided into a number of intervals or subsections where the boundary condition of equation (1) and (2) is to be applied. The lengths

of the intervals need not all be equal. For certain types of problems it could be advantageous to weight the lengths of the intervals. In order to keep the formulation as simple as possible, however, the present solution uses N-1 intervals of equal lengths and two intervals of half the length of the N-1 intervals. These half subsections were to allow a match point to be placed at the ends of the antenna and one on the bend of the antenna. In addition to these piecewise functions is another family of expansion functions which are commonly called entire-domain expansions. A discussion of the use of these entire-domain functions will not be attempted in this thesis.

All of the calculations and results presented in this thesis are obtained by the method of subsections in conjunction with point matching. In particular a series of piecewise constant and piecewise sinusoids are used to represent the unknown currents. The resulting equations are forced to be satisfied at a set of discrete points called match points. Using Dirac delta functions as weighting functions effectively requires that equation (24) be satisfied at N points on the contour of the antenna.

Harrington and Mautz [Ref. 2] have solved the single straight wire problem using three different procedures, all within the scope of the method of moments. In order of increasing computational complexity, these include:

(1) Point matching with pulses as basis functions

(2) Point matching with piecewise linear basis functions

(3) Galerkin's procedure with piecewise linear basis functions.

As the number of subsections increases, the second solution converges about twice as fast as the first. With antenna segments less than $\lambda/10$

in length, no significant difference in results is observed between Galerkin's procedure and the second procedure. Using piecewise sinusoids is equivalent to the second method in speed of convergance and and accuracy, but piecewise sinusoids do not require the segment length to be less than $\lambda/10$ in length to obtain this accuracy and speed.

Using Dirac delta functions as weighting functions gives

$$W_m = \delta(z - z_m)\hat{U}_z \qquad (33a)$$

$$W_s = \delta(s - s_m)\hat{U}_s \qquad (33b)$$

in which U_z is a unit vector directed along the positive z-axis and U_s is a unit vector directed along the positive s-axis. Using the piecewise constant subsection gives the following expressions for current

$$I(z) = \sum_{n=1}^{N} I_n P_n(z)\hat{U}_z \qquad (34a)$$

$$I(s) = \sum_{q=1}^{Q} I_q P_q(s)\hat{U}_s \qquad (34b)$$

where I_n and I_q are the complex constants to be determined and the pulse functions are constant over the increments. This statement in equation form is

$$P_n(z) = \begin{cases} 1 & z_n<z<z_{n+1} \\ 0 & \text{elsewhere} \end{cases} \qquad (35a)$$

$$P_q(s) = \begin{cases} 1 & s_q<s<s_{q+1} \\ 0 & \text{elsewhere} \end{cases} \qquad (35b)$$

Similarly, the piecewise sinusoid expression gave the following current equations

$$I(z) = \sum_{n=1}^{N} \frac{I_n \sin k(z'_{n+1}-z') + I_{n+1} \sin k(z'-z'_n)}{\sin k(z'_{n+1}-z'_n)} P_n(z)\hat{U}_z \qquad (36a)$$

$$I(s) = \sum_{q=1}^{Q} \frac{I_q \sin k(s'_{q+1}-s') + I_{q+1} \sin k(s'-s'_q)}{\sin k(s'_{q+1}-s'_q)} P_q(s)\hat{U}_s . \quad (36b)$$

C. LOCATION OF MATCH POINTS

There has been little quantitative study done on the precise effect of the location of the match points on the solution; however, it has been the experience of this author that the location of these match points significantly affects the accuracy and convergence of the solution. The number of match points and their location depends on such factors as the form of the integral equations (Pockington or Hallén), the type of current expansion used (pulse, sinusoids) and the allowable time and core for computation.

Extensive calculations have been carried out [Ref. 22] where the match points are located at ends of each current subsection. The results of these calculations indicate the solutions are quite stable and reasonably accurate even for problems where a large number of match points is used.

Other investigations have indicated that a good choice for the location of the match points is at the center of a current subsection. As the author discovered these conclusions have to be tempered with other information. One question is how and what kind of experimental verification is used. If a scattering cross section is used, the demonstrated results may differ from those obtained for an excited antenna. The reason is the cross section calculations effectively average the source and current distributions on the structure. Thus, a convergence study pertaining to input admittance, using scattered results, may be somewhat

inconclusive since the segmentation and match point dependence are directly related to source model and numerical convergence.

All of the results presented in this thesis are the result of locating the match points at the center of a current subsection for piecewise constant expansion and at the ends of the current subsections for piecewise sinusoid expansion.

V. APPLICATION OF THIN WIRE THEORY TO AN ARBITRARILY BENT WIRE MONOPOLE

A. GENERAL CONSIDERATIONS

A general theory for determining the currents induced on an arbitrarily bent wire monopole has been outlined. The purpose of this section is to illustrate how to obtain the exact solution. The configuration to be studied is composed of two straight nonorthogonal elements intersecting at a single junction which are driven above a perfect infinite ground plane. The problem is developed in a manner which allows general variations in many of the fundamental parameters. For example the lengths of the individual elements may vary, the angle between the individual elements may vary and the radii of the elements may vary. It should be noted that all variations in parameters are still subject to the basic assumptions and restrictions imposed by thin wire theory. These restrictions do not greatly inhibit the usefulness of the investigation since much general knowledge can be obtained from the thin wire approach.

B. COUPLED INTEGRAL EQUATIONS

The coupled integral equations for magnetic potential which were developed in a previous section are

$$A_z(z) = a_z \cos kz + b_z \sin kz + \frac{jk}{\omega} \int_{n=z_o}^{z} E_z(n) \sin k(z-n)dn$$

$$- \frac{1}{k} \int_{n=z_o}^{z} [\frac{\partial}{\partial y} A_y(n)] \cos k(z-n)dn \qquad (37a)$$

$$A_s(s) = a_s \cos ks + b_s \sin ks + \frac{jk}{\omega} \int_{n=s_o}^{s} E_s(n) \sin k(s-n)dn$$

$$- \frac{1}{k} \int_{n=s_o}^{s} \{\frac{\partial}{\partial w}[A_w(n)]\} \cos k(s-n)dn \tag{37b}$$

and the scalar potential equations are

$$- \frac{jk^2}{\omega} \phi^z(z) = -ka_z \sin kz + kb_z \cos kz + k \int_{n=z_o}^{z} \{\frac{\partial}{\partial y}[A_y(n)]\} \sin k(z-n)dn \tag{38a}$$

and

$$- \frac{jk^2}{\omega} \phi^z(z) = -ka_s \sin ks + kb_s \cos ks + k \int_{n=s_o}^{s} \{\frac{\partial}{\partial w}[A_w(n)]\} \sin k(s-n)dn. \tag{38b}$$

The constants a_z, b_z, a_s, and b_s must assume values which render equations (37) and (38) in compliance with the following boundary conditions:

(1) The scalar potential, ϕ, must be continuous along the wire segments except at the delta gap generator and in particular the scalar potential must be continuous at the wire junction.

(2) The magnetic potentials, A_s and A_z, must be continuous along the wire segments and at the wire junction.

(3) Current must vanish at the free end of the wire.

(4) The axial currents must satisfy Kirchoff's current law at the wire junction.

The magnetic potentials, A_s and A_z, developed earlier, are,

$$A_z(z) = \frac{\mu}{4\pi} \int_{z'} I_z(z') \frac{e^{-jkR}}{R} dz' \tag{39a}$$

and

$$A_s(s) = \frac{\mu}{4\pi} \int_{s'} I_s(s') \frac{e^{-jkP}}{P} ds' \tag{39b}$$

where $R = |\overline{r} - \overline{r}'|$ and $P = |\overline{\rho} - \overline{\rho}'|$. R and P were evaluated at $\overline{r}'$ and $\overline{\rho}'$ equal to the radius of the respective wire segments and ϕ' equal to zero. Using these values for $|\overline{r} - \overline{r}'|$ and $|\overline{\rho} - \overline{\rho}'|$ and assuming the surface current flows as an equivalent line current along the s and z axes is equivalent to using the reduced kernel. This will be proven in Appendix A along with conditions when the approximation is no longer valid.

Looking back at the development of equation (37) it is seen that the mathematical formulation was given without any physical insight as to where in space the equation was valid. In going from equations (10) and (11) to equation (18) there is a built-in assumption not expressly stated. This assumption is that the radius vector is fixed at some value which results in equation (37) being valid only along a line parallel to z and s directed wires. This result will be fully shown in Appendix B. Since equations (37) are valid along any line parallel to the z and s directed segments, one can require them to be along the surface of the wire segments which is along a line (a,o,z) and (a,o,s) respectively. This surface is also where the tangential E field is zero. Since the source points and field points could not lie along the same line, the reduced kernel has to be used with its equivalent line current flowing along the axis of the two wire segments.

Figure 1 defined the co-ordinate system which is composed of two co-ordinate subsystems. They are the (X,Y,Z) and (P,W,S) co-ordinate systems. The kernels for the wire segments are then expressed in these co-ordinate systems.

$$K_z(x,y,z,z') = \frac{e^{-jkR^z}}{R^z} \qquad K_z^i(x,y,z,z_i') = \frac{e^{-jkR_i^z}}{R_i^z} \tag{40a}$$

$$K_s(x,y,z,s') = \frac{e^{-jkR^s}}{R^s} \qquad K_s^i(x,y,z,s_i') = \frac{e^{-jkR_i^s}}{R_i^s} \tag{40b}$$

$$\hat{U}_w = -\hat{U}_z \sin\theta + \hat{U}_y \cos\theta$$
$$\hat{U}_s = \hat{U}_z \cos\theta + \hat{U}_y \sin\theta$$
$$\hat{U}_y = \hat{U}_s \sin\theta + \hat{U}_w \cos\theta$$
$$\hat{U}_z = \hat{U}_s \cos\theta - \hat{U}_w \sin\theta$$

$$\hat{U}_{s_i} = -\hat{U}_s \cos 2\theta + \hat{U}_w \sin 2\theta$$

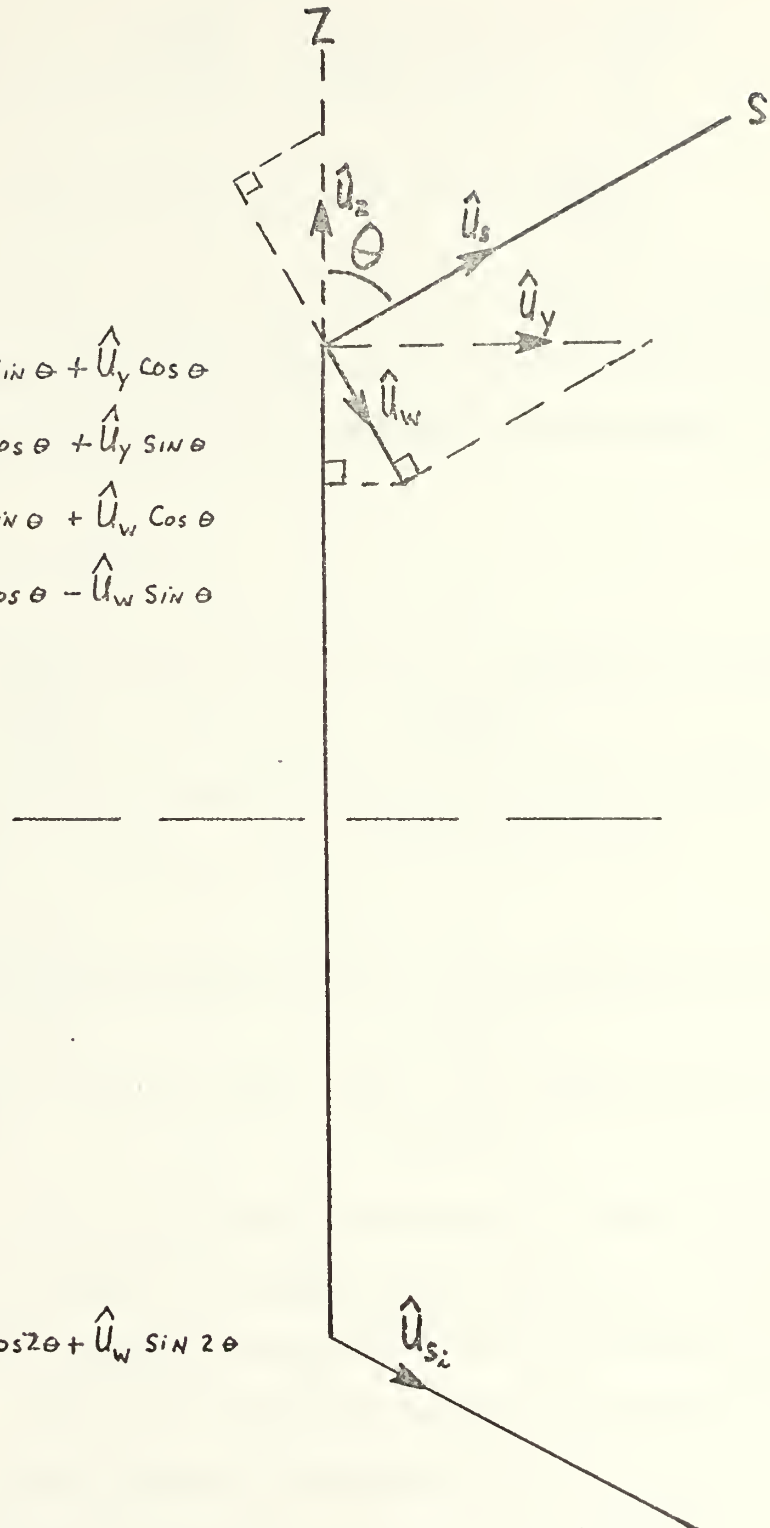


Figure 2. Antenna Coordinate Systems

$$G_z(p,w,s,z') = \frac{e^{-jkr^z}}{r^z} \qquad G_z^i(p,w,s,z_i') = \frac{e^{-jkr_i^z}}{r_i^z} \tag{40c}$$

$$G_s(p,w,s,s') = \frac{e^{-jkr^s}}{r^s} \qquad G_s^i(p,w,s,s_i') = \frac{e^{-jkr_i^s}}{r_i^s} \tag{40d}$$

$$R = \left\{(x - x')^2 + (y - y')^2 + (z - z')^2\right\}^{\frac{1}{2}}$$

$$r = \left\{(p - p')^2 + (w - w')^2 + (s - s')^2\right\}^{\frac{1}{2}} \text{ , in general.}$$

In order to account for the images due to a perfect ground plane one must include the image current kernels which are denoted above by the superscript i.

With reference to figure 2,

$$A(\bar{r}) = \hat{U}_z\left\{\int_{z'=0}^{h_z} I_z(z')K_z(x,y,z,z')dz' + \int_{z_i'=-h_z}^{0} I_z(z_i')K_z^i(x,y,z,z_i')dz_i'\right\}$$
$$+ \hat{U}_s \int_{s'=0}^{h_s} I_s(s')K_s(x,y,z,s')ds' - \hat{U}_{si} \int_{s_i'=-h_s}^{0} I_s(s_i')K_s^i(x,y,z,s_i')ds_i' \tag{41}$$

and

$$A(\bar{\rho}) = \hat{U}_z\left\{\int_{z'=0}^{h_z} I_z(z')G_z(p,w,s,z')dz' + \int_{z_i'=-h_z}^{0} I_z(z_i')G_z^i(p,w,s,z_i')dz_i'\right\}$$
$$+ \hat{U}_s \int_{s'=0}^{h_s} I_s(s')G_s(p,w,s,s')ds' - \hat{U}_{si} \int_{s_i'=-h_s}^{0} I_s(s_i')G_s^i(p,w,s,s_i')ds_i'. \tag{42}$$

When the $\hat{U}_{si}$ and $\hat{U}_s$ unit vectors are broken into their respective $\hat{U}_z$ and $\hat{U}_y$ components, they give the required $A_z(r)$ and $A_y(r)$. Similarly breaking $\hat{U}_z$ and $\hat{U}_{si}$ into their respective $\hat{U}_s$ and $\hat{U}_w$ components gives $A_s(\rho)$ and $A_w(\rho)$. These four magnetic potentials are needed in equation 37. These four required magnetic potentials are:

$$\frac{4\pi}{\mu} A_z(r) = \int_{z'=0}^{h_z} I_z(z')k^z(x,y,z,z')dz' + \cos\theta \int_{s'=0}^{h_s} I_s(s')k^s(x,y,z,s')ds', \quad (43a)$$

$$\frac{4\pi}{\mu} A_y(r) = \sin\theta \int_{s'=0}^{h_s} I_s(s')k^y(x,y,z,s')ds', \quad (43b)$$

$$\frac{4\pi}{\mu} A_s(p) = \int_{s'=0}^{h_s} I_s(s')\{G_s(p,w,s,s')ds' + \cos\theta\ G_s^i(p,w,s,s')\}ds' +$$

$$\cos\theta \int_{z'=0}^{h_z} I_z(z')G^z(p,w,s,z')dz' \quad (43c)$$

and

$$\frac{4\pi}{\mu} A_w(p) = -\sin 2\theta \int_{s'=0}^{h_s} I_s(s')G_s^i(p,w,s,s')ds' -$$

$$\sin\theta \int_{z'=0}^{h_z} I_z(z')G^z(p,w,s,z')dz'. \quad (43d)$$

Placing equations (43) into equations (37) gives

$$\frac{\mu}{4\pi} \int_{z'=0}^{h_z} I_z(z')K^z(x,y,z,z')dz' + \cos\theta \int_{s'=0}^{h_s} I_s(s')K^s(x,y,z,s')ds' = a_z\cos kz +$$

$$b_z\sin kz + \frac{jk}{\omega}\int_{n=z_o}^{z} E_z(n)\sin k(z-n)dn - \frac{1}{K}\int_{n=z_o}^{z} \{\frac{\partial}{\partial y}[A_y(n)]\} \cos k(z-n)dn \quad (44a)$$

and

$$\int_{s'=0}^{h_s} I_s(s')\{G_s(p,w,s,s') + \cos\ G_s^i(p,w,s,s')\}ds' + \cos\theta \int_{z'=0}^{h_z} I_z(z')G^z(p,w,s,z')dz' =$$

$$a_s\cos ks + b_s\sin ks + \frac{jk}{\omega}\int_{n=s_o}^{s} E_s(n)\sin k(s-n)dn - \frac{1}{K}\int_{n=s_o}^{s} \{\frac{\partial}{\partial w}[A_w(n)]\}\cos k(s-n)dn. \quad (44b)$$

Examination of equations (44) indicated that the partial derivatives of $A_y(r)$ and $A_w(\rho)$ are needed. The results for $A_y(r)$ will be shown and $A_w(\rho)$ will follow in a similar manner.

The form of $A_y(r)$ is

$$A_y(r) = \frac{e^{-jkR}}{R} \tag{45a}$$

and
$$\frac{\partial A_y(r)}{\partial y} = \frac{\partial A_y(r)}{\partial R}\frac{\partial R}{\partial y}$$

$$\frac{\partial A_y(r)}{\partial y} = -\frac{(1 + jkR)(y - y')e^{-jkR}}{R^3} \quad . \tag{45b}$$

Similarly, for $A_w(\rho)$,

$$\frac{\partial A_w(\rho)}{\partial w} = -\frac{(1 + jkr)(w - w')e^{-jkr}}{r^3} \quad . \tag{46}$$

Placing equations (45) and (46) into equations (44) and examining the results closely, it is seen that the double integrals have an inner integral of the form

$$\int [\frac{jk}{R^2} + \frac{1}{R^3}]e^{-jkR} \cos k(\beta-n)dn \tag{47}$$

where
$$R = \left\{a^2 + g(\alpha) + [n - f(\alpha)]^2\right\}^{\frac{1}{2}}$$

and f and g are any function of the parameter α. Reference [23] gave the exact solution for this integral which reduced the double integral to a single integral and thereby greatly reducing the computer integration time and numerical errors. The exact derivation will be shown in Appendix C. Now place the result of Ref. [23] into equation (44a). With rearrangement of equation (44a) the following equation is obtained:

$$[\frac{4\pi}{\mu}]\ a_z \cos kz + [\frac{4\pi}{\mu}]\ b_z \sin kz + [\frac{4\pi jk}{\mu\omega}] \int_{n=z_o}^{z} E_z(n) \sin k(z-n)dn -$$

$$\frac{4\pi}{\mu k} \int_{s'=s_o}^{h_s} I_s(s') \left(K^y(x,y,n,s')\left\{\frac{[n-f(s')]\cos k(z-n) - jR\sin k(z-n)}{R(R^2 - [n-f(s')]^2)}\right\}\Bigg|_{n=z_o}^{z}\right)ds' -$$

$$\int_{z'=z_o}^{h_z} I_z(z')K^z(x,y,z,z')dz' - \cos\theta \int_{s'=s_o}^{h_s} I_s(s')K^s(x,y,z,s')ds' = 0 \qquad (48)$$

Similarly, for equation (44b):

$$[\frac{4\pi}{\mu}]\ a_s \cos ks + [\frac{4\pi}{\mu}]\ b_s \sin ks + [\frac{4\pi jk}{\mu\omega}] \int_{n=s_o}^{s} E_s(n) \sin k(s-n)dn +$$

$$\frac{4\pi \sin 2\theta}{\mu} \int_{s'=s_o}^{h_s} I_s(s') \left(G^i(p,w,n,s')\left\{\frac{[n-f(s')]\cos k(s-n) - jr\sin k(s-n)}{r(r^2 - [n-f(s')]^2)}\right\}\Bigg|_{n=s_o}^{s}\right)ds' +$$

$$\frac{4\pi \sin \theta}{\mu} \int_{s'=s_o}^{h_s} I_s(s') \left(G^z(p,w,n,z')\left\{\frac{[n-f(z')]\cos k(s-n) - jr\sin k(s-n)}{r(r^2 - [\ -f(z')]^2)}\right\}\Bigg|_{n=s_o}^{s}\right)ds' -$$

$$\int_{s'=s_o}^{h_s} I_s(s')[G_s(p,w,s,s') - \cos G_s^i(p,w,s,s')]ds' -$$

$$\cos\theta \int_{z'=z_o}^{z} I_z(z')G^z(p,w,s,z')dz' = 0 \quad . \qquad (49)$$

For a structure driven by a delta gap source of emf, V, on an element at a location displaced g units above a ground plane the following equations are obtained [Ref. 20]:

$$V_z(z) = \frac{jk}{\omega} \int_{n=z_o}^{z} -V\delta(n-g) \sin k(z-n)dn \qquad (50a)$$

$$V_z(z) = -\frac{jk}{\omega}[V \sin k(z-g)\ U(z-g)]. \qquad (50b)$$

It is desirable to obtain a representation of the emf when the structure is base driven, i.e., when g is zero. A structure above a ground plane, regardless of the nature of its excitation, always gave rise to zero scalar potential everywhere on the ground plane. This is a direct result of the uniqueness theorem and boundary conditions at a perfect conducting surface. Thus, it is necessary that, if the element and its source are to be equivalent, the impressed emf must be symmetric; otherwise, the currents and charges in a free-space "equivalent" of the structure above a ground would not satisfy imagy theory requirements.

It is from this discussion that the modification to equation (50b) is found. When a base-driven structure is being analyzed, one sets g equal to zero and V equal to V/2. If V/2 were not used in place of V, one would be treating an antenna driven by 2V when the free-space "equivalent" structure is formed. These results are

$$V_z(z) = -\frac{jk}{\omega}\frac{V}{2}\sin kz \ . \tag{51}$$

Since the s-directed segment is not driven by a separate generator, this gives

$$V_s(s) = 0 \text{ or } \frac{jk}{\omega}\int_{n=s_o}^{s}[0]\sin k(s-n)dn = 0 \ . \tag{52}$$

Now the boundary conditions are applied. Due to symmetry of the charges on the structure and its image, the scalar potential must be zero at the point where the vertical element enters the ground plane. This implies

$$-\frac{jk^2}{\omega}\phi^z(0) = 0 \tag{53a}$$

which gives $b_z = 0$ (53b)

Next by condition one the scalar potential must be continuous at the wire junction which gives

$$\phi^z(h_z) = \phi^s(0) \tag{54a}$$

and results in

$$-ka_z \sin kh_z - kb_s + k\int_{n=z_o}^{z} \{\frac{\partial}{\partial y}[A_y(n)]\} \sin k(z-n)\,dn - \frac{jk}{\omega}\frac{V}{2}\cos kh_z = 0 \tag{54b}$$

Conditions three and four require

$$I_Q^S = 0 \quad \text{and} \tag{55}$$

$$I_N^Z = I_1^S\ . \tag{56}$$

Clearing all constants and applying all boundary conditions, the following equations for $I_z(z)$ and $I_s(s)$ are obtained:

$$[\frac{4\pi}{\mu}]a_z \cos kz - \int_{s'=0}^{h_s} I_s(s')\Big(K^y(x,y,n,s')\{\frac{[n-f(s')]\cos k(z-n) - jR\sin k(z-n)}{R(R^2 - [n-f(s')]^2)}\}\Big|_{n=0}^{z}\Big)ds'$$
$$- \frac{jV}{60}\sin kz - \int_{z'=0}^{h_z} I_z(z')K^z(x,y,z,z')dz' - \cos\theta \int_{s'=0}^{h_s} I_s(s')K^s(x,y,z,s')ds'$$
$$= 0 \tag{57a}$$

$$[\frac{4\pi}{\mu}]a_z \cos ks + \int_{s'=0}^{h_s} I_s(s')\Big(G^i(p,w,n,s')\{\frac{[n-f(s')]\cos k(s-n) - jr\sin k(s-n)}{r(r^2 - [n-f(s')]^2)}\}\Big|_{n=0}^{s}\Big)ds'$$
$$+[\frac{4\pi}{\mu}]b_s \sin ks + \int_{z'=0}^{h_z} I_z(z')\Big(G^z(p,w,n,z')\{\frac{[n-f(z')]\cos k(s-n) - jr\sin k(s-n)}{r(r^2 - [n-f(z')]^2)}\}\Big|_{n=0}^{s}\Big)dz'$$
$$- \int_{s'=0}^{h_s} I_s(s')G_s(p,w,s,s')ds' - \cos\theta \int_{s'=0}^{h_s} I_s(s')G_s^i(p,w,s,s')ds'$$
$$- \cos\theta \int_{z'=0}^{h_z} I_z(z')G^z(p,w,s,z')dz' = 0 \tag{57b}$$

and

$$- [\frac{4\pi}{\mu}] a_z \sin kh_z + \int_{s'=0}^{h_s} I_s(s') \left(K^y(x,y,n,s') \left\{ \frac{[n-f(s')]\cos k(h_z-n) - jR\sin k(h_z-n)}{R(R^2 - [n-f(s')]^2)} \right\} \Big|_{n=0}^{h_z} \right) ds'$$

$$- [\frac{4\pi}{\mu}] b_s - \frac{jV}{60} \cos kh_z = 0 \tag{57c}$$

$$I_Q^S = 0 \tag{57d}$$

$$I_N^Z = I_1^S . \tag{57e}$$

There are five unknowns a_s, b_s, a_z, $I_z(z')$, and $I_s(s')$ to be found and five equations which relate these quantities; hence, there is a unique solution to this system of equations for the five unknowns. [Ref. 20]. Obtaining the current distributions, all the other desired quantities such as electric field, input impedance can be determined.

C. NUMERICAL SOLUTION

The system of integral equations given in the previous section must be solved in order to determine the unknown current distributions. The direct integration technique as outlined earlier is used to effect a numerical (discrete) solution for the current distributions.

The first step in the direct integration technique is the choice of the expansion functions and the testing functions. As pointed out in an earlier section, the method of subsections in conjunction with point matching is a very convenient approach. The testing functions are Dirac delta functions since point matching is used.

The expansion functions for the unknown currents are chosen to be piecewise constant for its ease of use and piecewise sinusoids for their improvement in convergence and required computation time. Symbolically,

the piecewise constant function is represented by

$$I_z(z') = \sum_{n=1}^{N} I_n^z [U(z' - z_n') - U(z' - z_{n+1}')] \tag{58a}$$

$$I_s(s') = \sum_{q=1}^{Q} I_q^s [U(s' - s_q') - U(s' - s_{q+1}')] \tag{58b}$$

and where $U(\alpha)$ is the familiar unit step function

$$U(\alpha) = \begin{cases} 1 & \alpha > 0 \\ 0 & \alpha < 0 \end{cases} . \tag{59}$$

Consistent with the above description, the current can be written as

$$I_z(z') = I_1^z U(z_2' - z') + \sum_{n=2}^{N-1} I_n^z [U(z'-z_n') - U(z'-z_{n+1}')] + I_N^z U(z'-z_N') \tag{60a}$$

and

$$I_s(s') = I_1^z U(s_2' - s') + \sum_{q=2}^{Q-1} I_q^s [U(s'-s_q') - U(s'-s_{q+1}')] + I_Q^s U(s'-s_Q') . \tag{60b}$$

Similarly, one can write for the piecewise sinusoids the following expressions:

$$I_z(z') = \sum_{n=1}^{N} \frac{I_n^z \sin k(z_{n+1}'-z') + I_{n+1}^z \sin k(z'-z_n')}{\sin k(z_{n+1}'-z_n')} \{U(z'-z_n') - U(z'-z_{n+1}')\} \tag{61a}$$

$$I_s(s') = \sum_{q=1}^{Q} \frac{I_q^s \sin k(s_{q+1}'-s') + I_{q+1}^s \sin k(s'-s_q')}{\sin k(s_{q+1}'-s_q')} \{U(s'-s_q') - U(s'-s_{q+1}')\} \tag{61b}$$

With the assumption that the segments were all equally spaced, the term $z_{n+1}'-z_n'$ is a constant. This allows a term delta (Δ) to be defined as

$$\Delta = \frac{1}{\sin k \text{ (constant)}} . \tag{62}$$

With this definition and equations (61) the current can be written as

$$I_z(z) = \frac{I_1^z}{\Delta} \sin k(z_2'-z')U(z_2'-z') + \frac{I_{N+1}^z}{\Delta} \sin k(z'-z_n')U(z'-z_N') + \qquad (63)$$

$$\frac{1}{\Delta}\sum_{n=2}^{N} I_n^z\left\{\sin k(z'-z_{n-1}')\left(U(z'-z_{n-1}')-U(z'-z_n')\right)+ \sin k(z_{n+1}'-z')\left(U(z'-z_n')-U(z'-z_{n+1}')\right)\right\}$$

and similarly for the s current. With these expansion functions, the current distribution appeared graphically as shown in Figures 3 - 9.

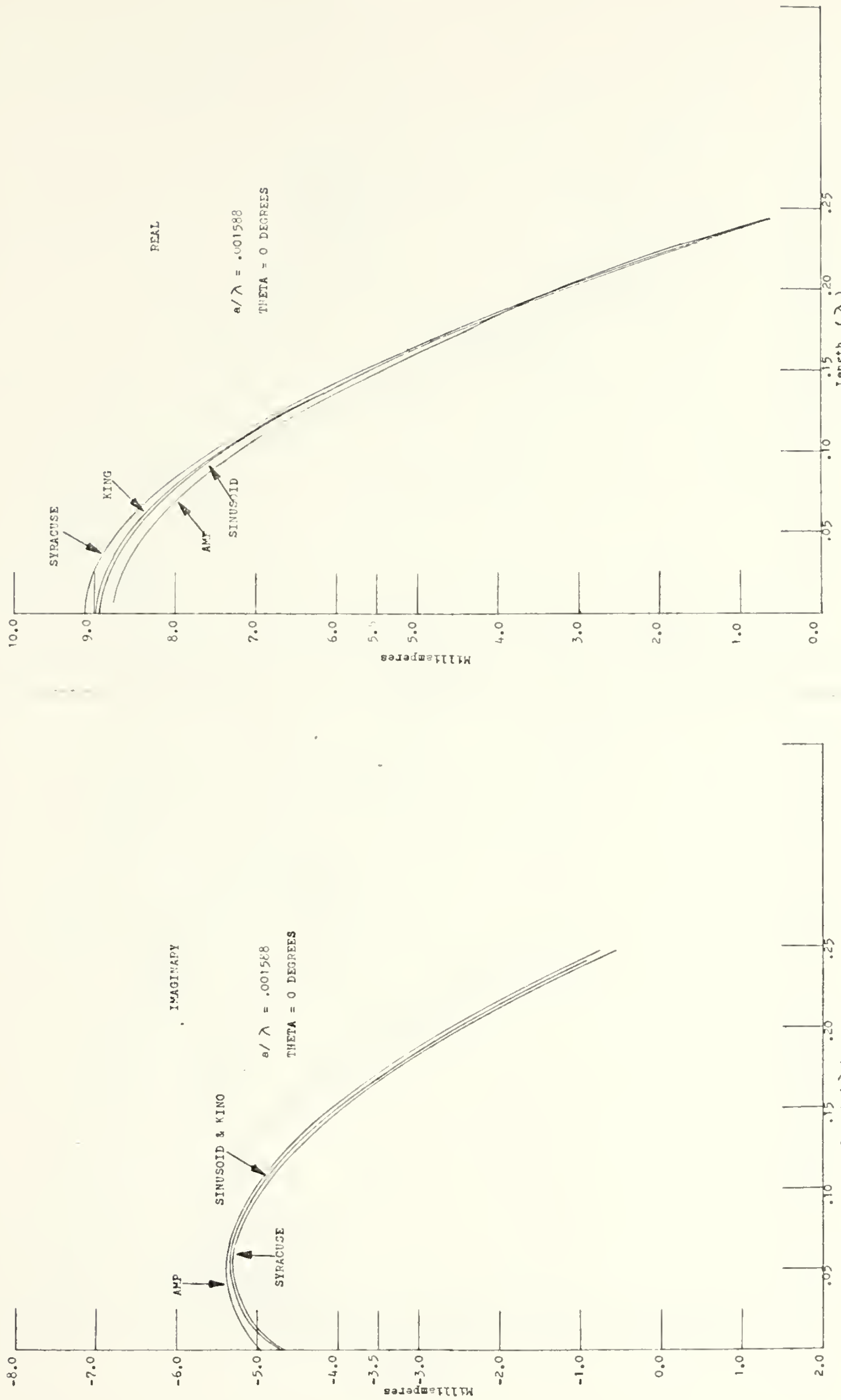


Figure 3. Real and imaginary parts of the current distributions for the radius = .001588λ, theta = 0°, 36 segments.

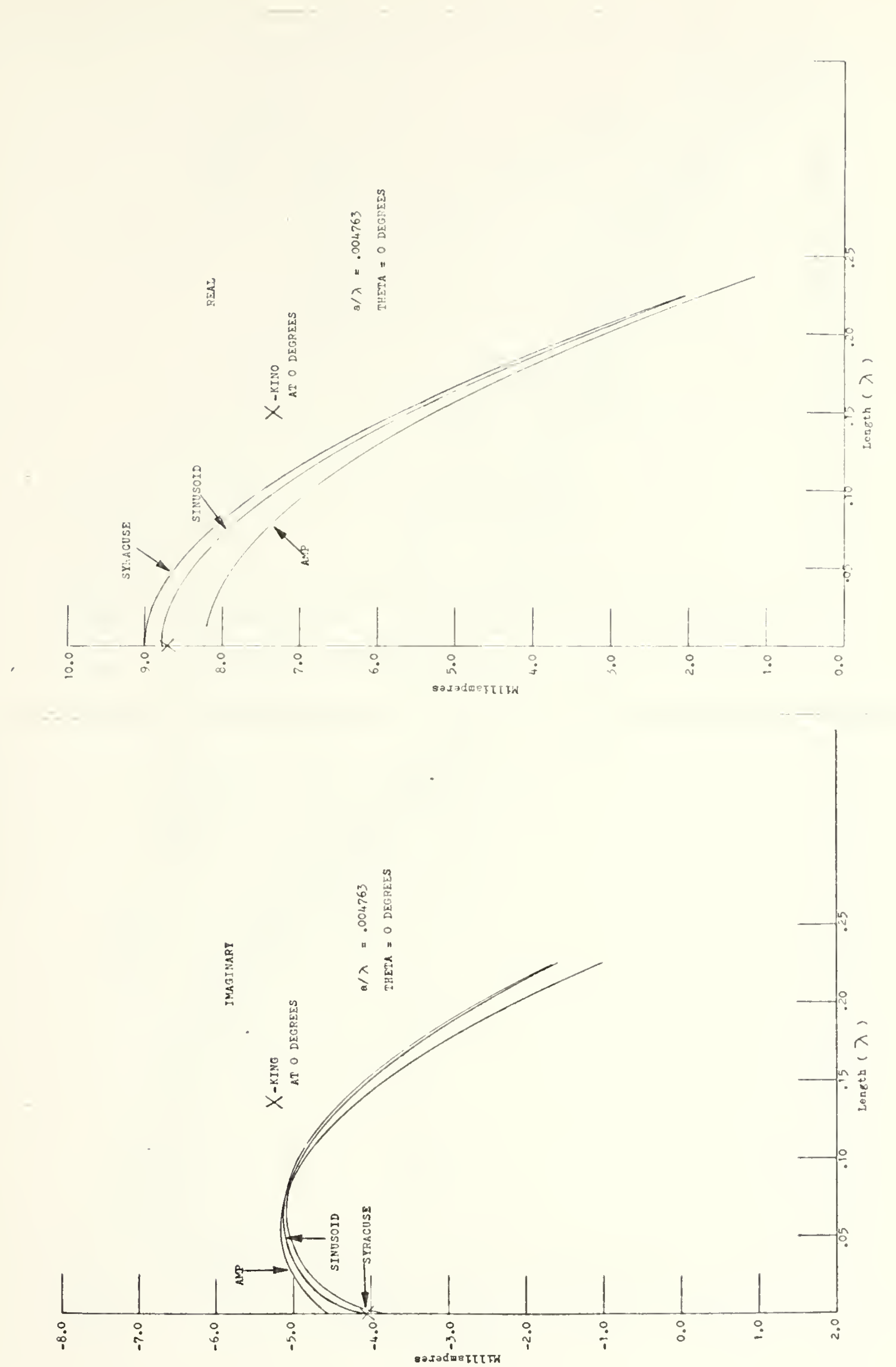
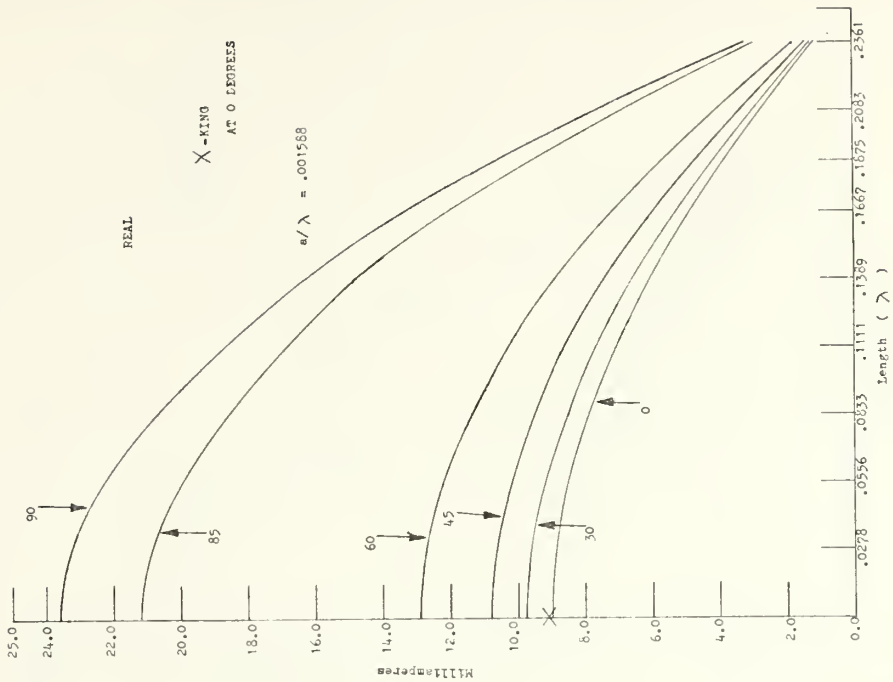


Figure 4. Real and imaginary parts of the current distributions for the radius = .004763λ, theta = 0°, 20 segments.



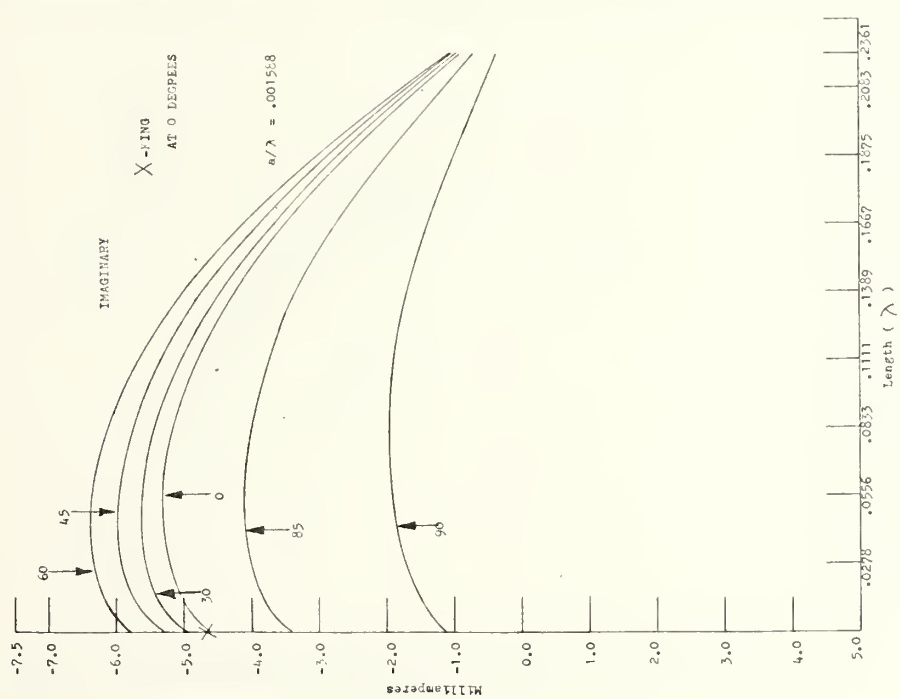
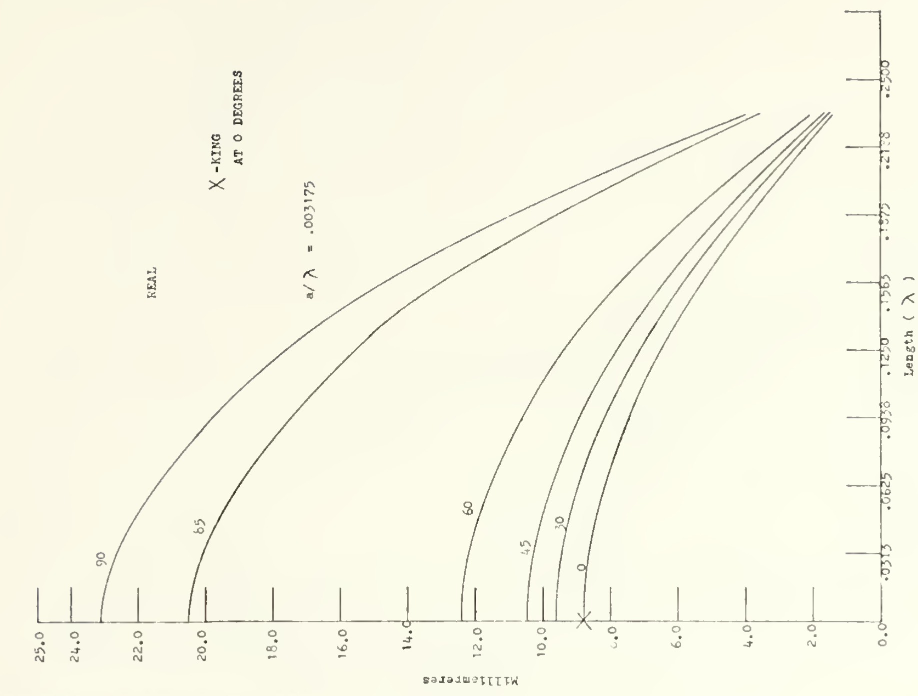


Figure 5. Real and imaginary parts of the current distributions for the radius = .001588λ, theta = 0°, 30°, 45°, 60°, 85°, 90°, 36 segments.



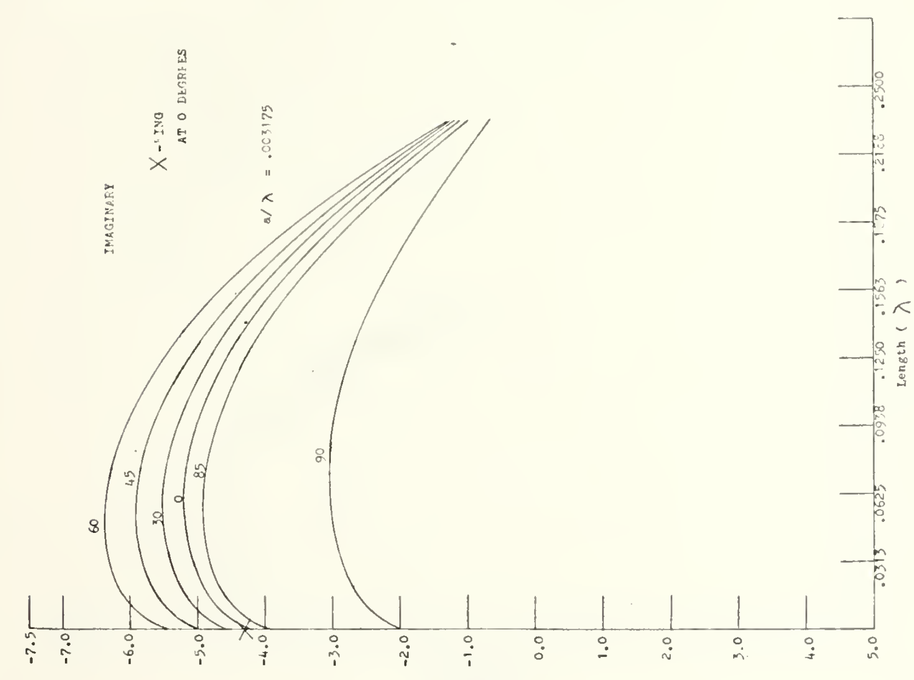


Figure 6. Real and imaginary parts of the current distribution for the radius = .003175λ, theta = 0°, 30°, 45°, 60°, 85°, 90°, 32 segments.

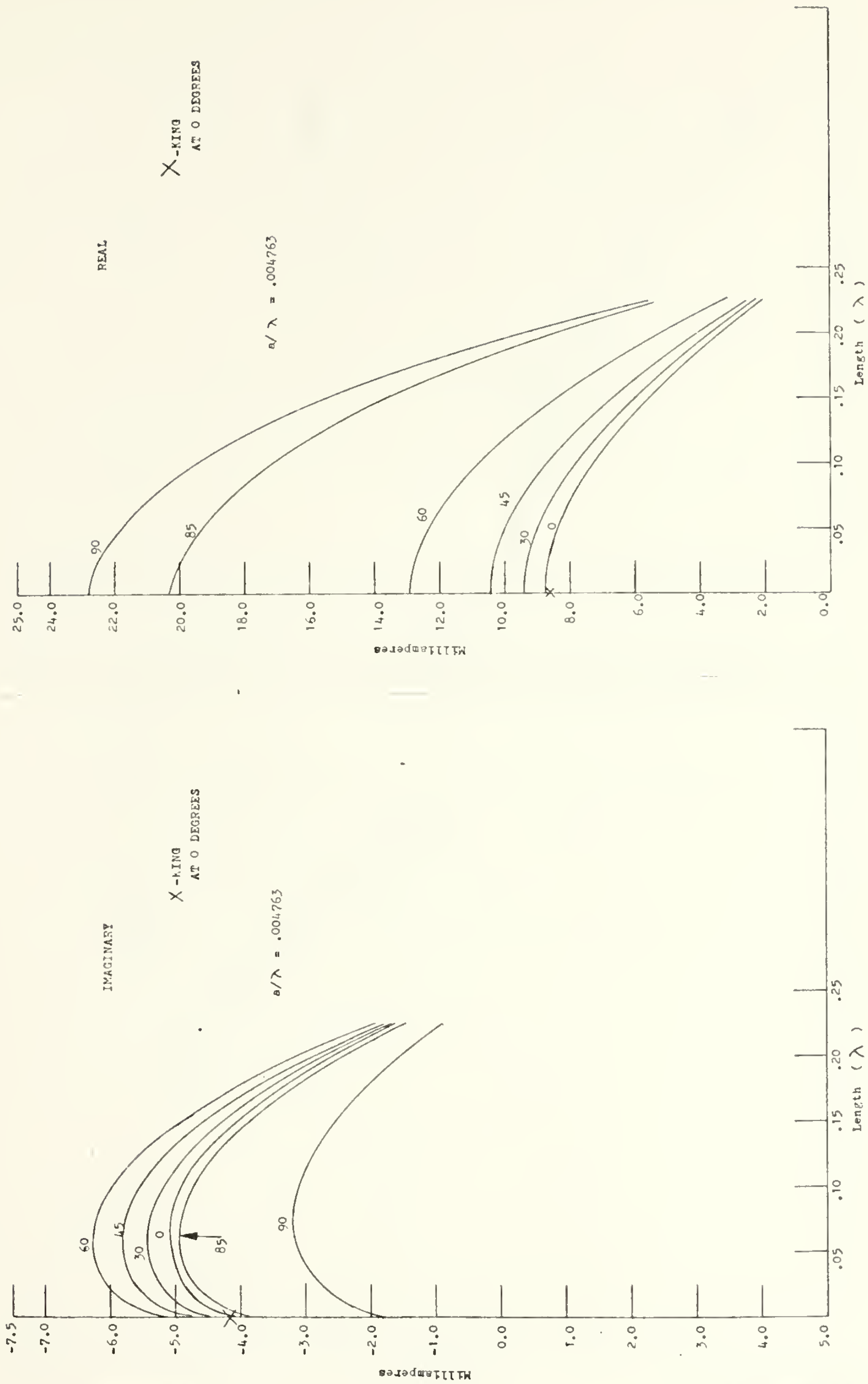


Figure 7. Real and imaginary parts of the current distribution for the radius = .004763λ, theta = 0°, 20°, 45°, 60°, 85°,90°, 25 segments.

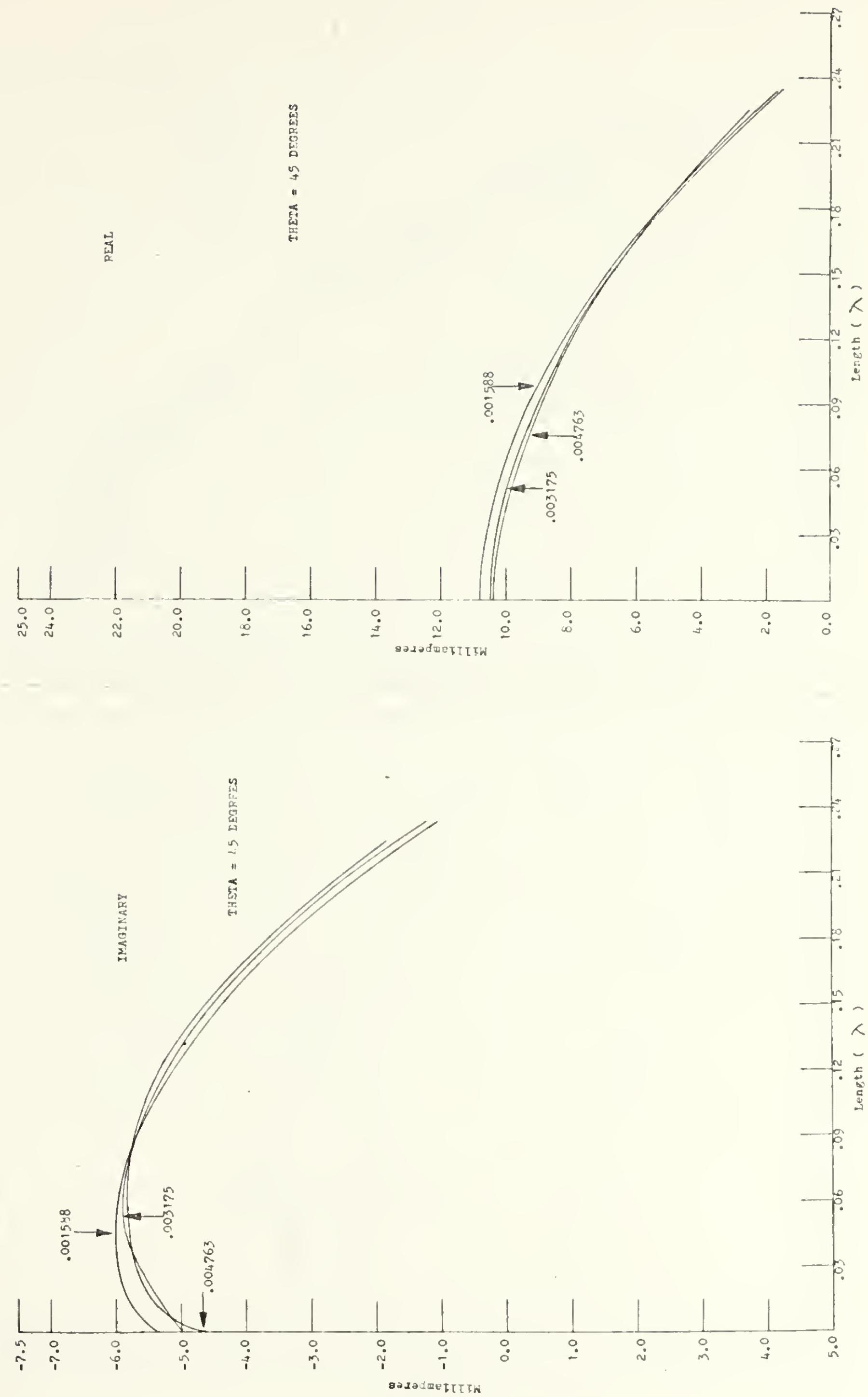


Figure 8. Real and imaginary parts of the current distributions for the radius = .001588λ, .003175λ, .004763λ, theta = 45°.

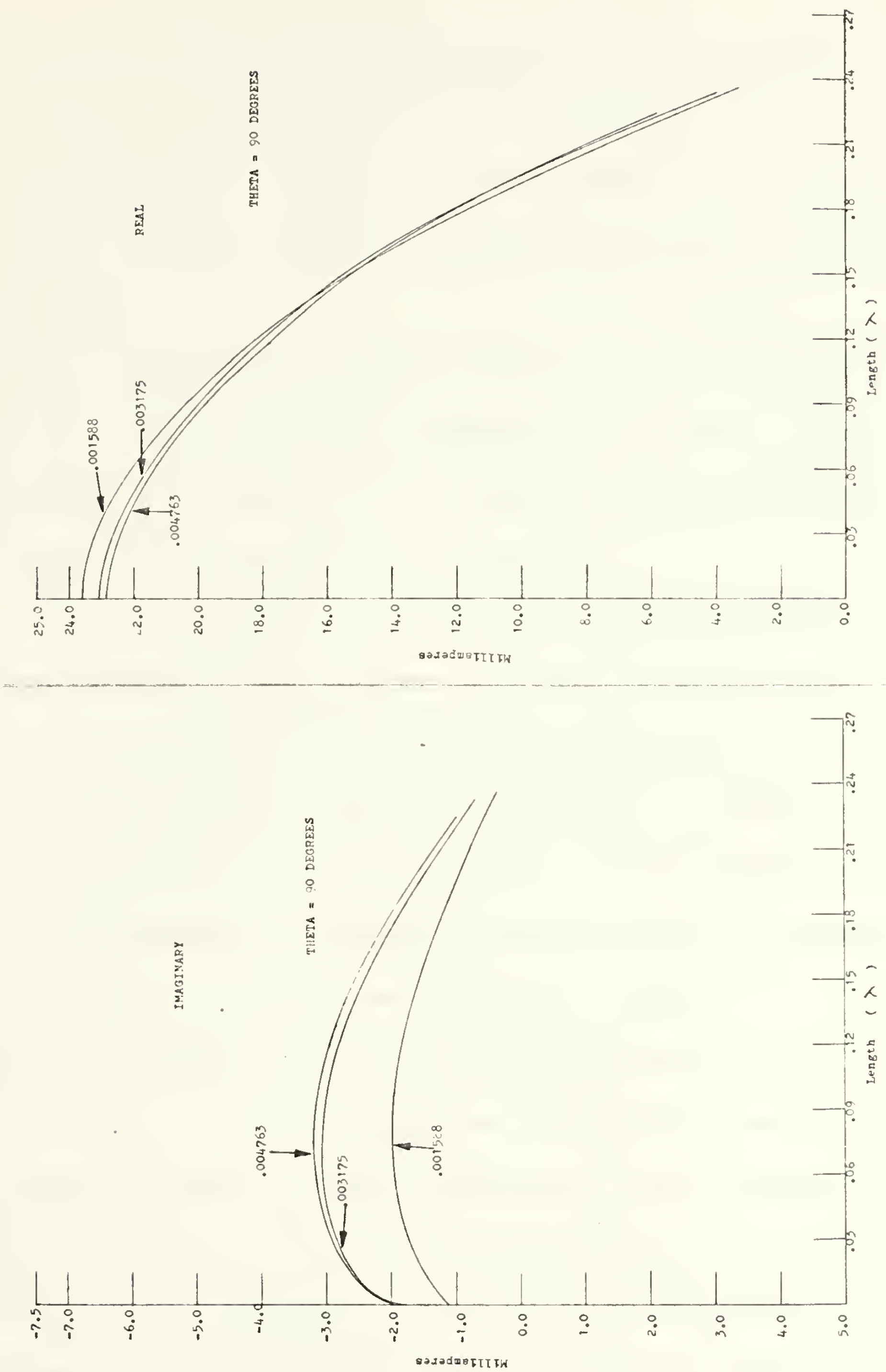


Figure 9. Real and imaginary parts of the current distributions for the radius = .001588λ, .003175λ, .004763λ, theta = 90°.

VI. RESULTS

The system of integral equations and the proposed solution techniques are quite general, and they allow variations in many of the parameters in the basic configuration. However, certain parameter variations appear to be more interesting than others. This study concentrates its efforts in three main areas:

1. The ability of the numerical solution to give the result obtained by King in Ref. 26 when the angle between the two wires is zero.

2. Current as a function of the angle between the two wires.

3. Current as a function of wire diameter.

The antenna is assumed to be driven by a delta gap generator. The current distributions are presented in terms of a dipole vice a monopole. The real and imaginary parts of the current are plotted for various bends of the antenna. The total length is $\lambda/4$ for all the figures with the antenna bent at $\lambda/8$ to give two equal lengths. Although these results are the only ones presented, the computer program allows the antenna to be bent at any point and the antenna to have any length.

Figures 3 and 4 show the current for two different radii at an angle of zero degrees for a comparison against King's frill feed in Ref. 26. These figures also show the result from Ref. 27 and 28 against King.

References 27 by D.C. Kuo and B.J. Strait at Syracuse University uses a modified Pockington formulation with thin-wire approximations. ϕ is used instead of $\overline{A}$ and derivatives are approximated by finite differences over the same intervals used for integration. Galerkin's procedure is used with triangle expansion functions which are non-zero only over four

consecutive segments. Successive triangles overlap every two segments except on the ends of open wires. Junctions are treated as open wires each having a two segment overlap at the junction. An extended operator is used because the first order derivatives of the triangle functions do not exist at certain points.

Reference 28 by MB Associates and called the Antenna Modelling Program uses a Pockington formulation with thin-wire approximations. Point matching is used with the current being expanded as a sine, cosine and constant with an interpolation procedure so only N linear equations need to be solved. The extrapolated current for a given segment is forced to match the center current values in two adjacent segments. This interpolation procedure is also applied to wire junctions.

Figures 5,6 and 7 give the currents for antennas of three different radii having 0, 30, 45, 60 and 90 degree bends. King's input values for a 0 degree bend are also plotted. Figures 8 and 9 show the currents for different radii antennas plotted on the same graph for bends of 45 and 90 degrees, respectively.

Several general comments concerning the data presented may prove helpful to the reader. All the results presented here are for the case of a one volt drive and assumes an open-ended perfectly conducting hollow cylindrical tube as the antenna. Extensive calculations have been carried out for bends of 0 to 90 degrees for each antenna radius given, but the plotting of all these angles on a single graph would obscure the overall view. It is believed that the data given is sufficient to give the reader an indication of the general variation of the current with respect to the parameter of interest.

VII. CONCLUSIONS

In this thesis a new method is presented for analysis of an arbitrarily bent monopole. The arbitrarily bent monopole is a good choice because it contains cross- coupling terms in the Hallén formulation which are not present in the 0 or 90 degree case. The currents had to be obtained using the appropriate boundary value problem based on Maxwell's equations which are formulated as a operator equation. The general procedure, "method of moments", is used to represent the operator by a matrix. Geometrical symmetries are utilized to simplify the matrix inversion.

Numerical results are compared with data obtained by different theoretical techniques and with experimental results where possible. Agreement is generally very good and this is interpreted as an indication of the accuracy of the solution.

Certain parameters should now be concentrated on if the conclusions are to be used properly. These efforts should be in these areas:

1. Choice of bases
2. Antenna drive
3. Discontinuity at the bend.

As can be seen by equation (36) the sinusoid current expansion equations are continuous but their derivatives are not. This implies mathematically that there are pockets of charge at the end of each subinterval which physically are not there. If one is interested in the near field, these mathematical charges cause discontinuities in the near field. One solution has been to use a least square fit to the current values obtained.

The delta gap drive was chosen because it was an ideal model. Even though the delta gap gives a physically meaningless infinite input susceptance associated with the knife edges, the answer is close to that achieved by experiment. The reason is that any method that approximates the current by a series of continuous functions will not reproduce the extremely sharp rise in amplitude near $z = z'$ unless a very large number of terms is used. A better drive would be the frill feed as given in reference [29] which corresponds closely to the actual drive used on physical antennas.

Although not plotted on the figures given, a discontinuity in current was obtained within one subinterval either side of the bend. This was to be somewhat expected because the formulation considers two straight wires joined at a point which gives a sharp bend where the cross section is abruptly changed. When one uses the approximate kernel, $I(z)$ must be a slowly varying function in the vicinity of $z = z'$ for the approximate kernel to be within an order of the square of the radius of the exact kernel. This is not true at the bend. It is this slow variation or fast variation that causes the various methods of iteration (Hallén, Gray, King-Middleton) to require more or fewer terms in representing the current.

APPENDIX A: REDUCED KERNEL

Equation (39) in the thesis must be investigated and R and P determined in a more precise manner. A detailed development of the vector potential must now be presented.

The vector potential due to a volume current density can be written as

$$\overline{A}(\overline{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\overline{J}(\overline{r}')e^{-jk|\overline{r}-\overline{r}'|} dV(\overline{r}')}{|\overline{r} - \overline{r}'|} \quad . \tag{1}$$

Looking at Figure 10, it is seen that the vector $\overline{r}$ locates the point at which $\overline{A}(\overline{r})$ is to be determined and $\overline{r}'$ locates the source point. With a symmetrically driven antenna whose wall thickness approaches zero, the vector potential has only a z component which can be written as

$$A_z(\overline{r}) = \frac{\mu}{4\pi} \int_{z'} \int_{\phi'=0}^{2\pi} \int_{r'=0}^{a} \frac{J_z(r',z')e^{-jk|\overline{r}-\overline{r}'|}}{|\overline{r} - \overline{r}'|} r'dr'd\phi'dz' \tag{2}$$

or integrating over r' gives

$$A_z(\overline{r}) = \frac{\mu}{4\pi} \int_{z'} \int_{\phi'=0}^{2\pi} \frac{[aI_z(a,z')]e^{-jk|\overline{r}-\overline{r}'|}}{|\overline{r} - \overline{r}'|} d\phi'dz' \quad . \tag{3}$$

Now due to symmetry

$$I(z') = 2\pi a J_z(a,z') \tag{4a}$$

or

$$aJ_z(a,z') = \frac{1}{2\pi} I(z') \quad . \tag{4b}$$

Substituting (4b) into equation (3) gives

$$A_z(r) = \frac{\mu}{4\pi} \int_{z'} I(z')\left\{\frac{1}{2\pi} \int_{\phi'=0}^{2\pi} \frac{e^{-jk|\overline{r}-\overline{r}'|}}{|\overline{r}-\overline{r}'|} d\phi'\right\}dz' \tag{5}$$

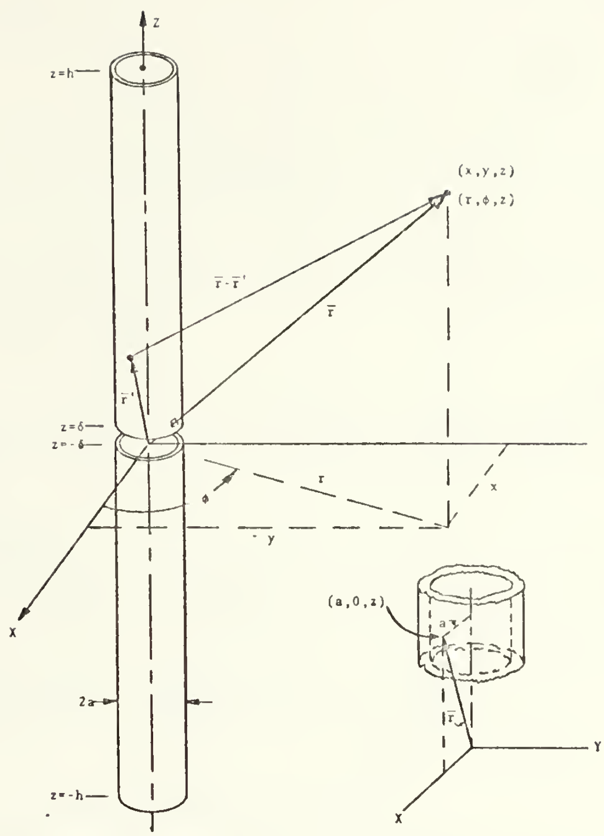


Figure 10. Antenna Geometry assuming a surface current in the z direction.

where r' is constrained to remain on the surface of the cylinder.

Appealing to basic vector algebra and trigonometry and being aided by Figure 10 one can write $\overline{r}$ and $\overline{r}'$ in the following form:

$$\overline{r} = x\hat{U}_x + y\hat{U}_y + z\hat{U}_z = r\cos\phi\hat{U}_x + r\sin\phi\hat{U}_y + z\hat{U}_z \tag{6a}$$

$$\overline{r}' = x'\hat{U}_x + y'\hat{U}_y + z'\hat{U}_z = r'\cos\phi'\hat{U}_x + r'\sin\phi'\hat{U}_y + z'\hat{U}_z \,. \tag{6b}$$

Now $|\overline{r} - \overline{r}'|$ is written as

$$|\overline{r} - \overline{r}'| = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{\frac{1}{2}}$$

$$= [r^2 + (r')^2 - 2rr'\cos(\phi - \phi') + (z - z')^2]^{\frac{1}{2}} \tag{7}$$

If $|\overline{r} - \overline{r}'|$ is evaluated at r' equal to the radius of the z-directed wire and ϕ equal to zero then

$$|\overline{r} - \overline{r}'|_{\substack{r'=a \\ \phi=0}} = [r^2 + a^2 - 2ra\cos\phi' + (z - z')^2]^{\frac{1}{2}} \tag{8}$$

Trigonometry gives

$$1 - \cos 2\beta = 2\sin^2\beta \tag{9}$$

Applying equation (9) to (8) with r = a gives

$$|\overline{r} - \overline{r}'| = [4a^2\sin^2(\tfrac{\phi'}{2}) + (z - z')^2]^{\frac{1}{2}} \,. \tag{10}$$

Going back to equation (5) and inserting equation (10) gives

$$A_z(a,z) = \frac{\mu}{4\pi}\int_{z'} I(z')\left\{\frac{1}{2\pi}\int_{\phi'=0}^{2\pi}\frac{e^{-jk[4a^2\sin^2(\frac{\phi'}{2}) + (z - z')^2]^{1/2}}}{[4a^2\sin^2(\frac{\phi'}{2}) + (z - z')^2]^{1/2}}d\phi'\right\}dz'. \tag{11}$$

One sees immediately that at z equal z' and ϕ' equal 0 or 2π the inner integrand is singular; however, the integrand is logarithmically singular and is integrable. Even though it is integrable, evaluation of the integrand by numerical techniques is extremely difficult. This is one major reason why a reduced, average or approximate $|\overline{r} - \overline{r}'|$ is found.

For $|z - z'| \gg a$ and for thin wires where $a \ll \lambda$ the following approximation can be obtained.

$$|\bar{r} - \bar{r}'| = [(a - a')^2 + (2a \sin [\frac{\phi'}{2}])^2]^{1/2} \doteq |z - z'| \qquad (12)$$

Even this for thin wires is not a good approximation where $z \doteq z'$. Observing that $2a \sin (\frac{\phi'}{2})$ is the distance from a point $(a,0,z')$ to a point (a,ϕ',z), it is seen that both points are on the same circle of radius a in a constant z' plane. Looking at Figure 10 it is seen that $2a \sin (\frac{\phi'}{2})$ was a distance that assumed values from 0 to 2a as ϕ' varied from 0 to 2π with a mean value of a. Placing the mean value into equation (11) and integrating gives

$$A_z(a,z) = \frac{\mu}{4\pi} \int_{z'} I(z') \frac{e^{-jk[a^2 + (z - z')^2]^{1/2}}}{[a^2 + (z - z')^2]^{1/2}} dz' . \qquad (13)$$

It is restated that equation (13) is valid only for $a \ll \lambda$ and $a \ll h$. Now going back and using Figure 11, equation (5), and assuming the current to be a filament along the z axis, the vector potential $A_z(\bar{r})$ at a point $(a,0,z)$ would be

$$A_z(a,0,z) = \frac{\mu}{4\pi} \int_{z'} I(z') \frac{e^{-jk[a^2 + (z - z')^2]^{1/2}}}{[a^2 + (z - z')^2]^{1/2}} dz' \qquad (14)$$

which is exactly the same equation as equation (13). This is a most important point. Using the reduced kernel is the same as treating surface current on the cylinder as if it were an equivalent line current along the cylinder axis.

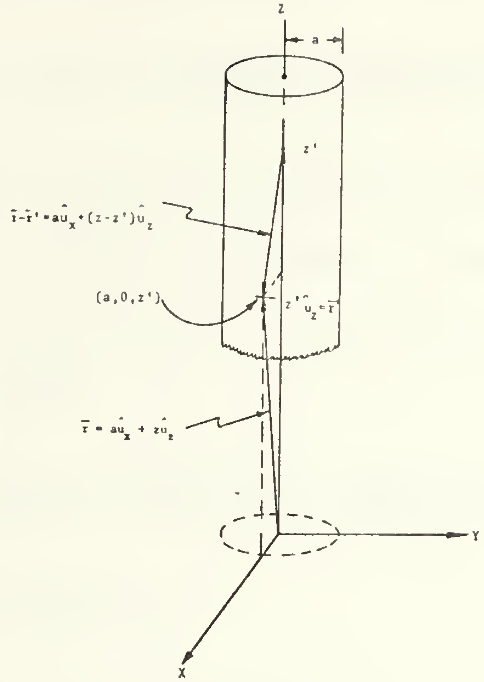


Figure 11. Antenna Geometry assuming a filamentary current along the z axis.

APPENDIX B: SOLUTION OF COUPLED INTEGRAL EQUATIONS

Equation (10) and (11) in the thesis give the two coupled partial differential equation that must be solved. Equation (12) gave the form that equation (10) and (11) must be in to apply the solution given in equation (17). Equation (10) gives the form for the z-directed wire as

$$\frac{\partial^2}{\partial z^2}[A_z(\bar{r})] + k^2 A_z(\bar{r}) = \frac{jk^2}{\omega} E_z^P(\bar{r}) - \frac{\partial}{\partial z}[\frac{\partial}{\partial y} A_y(\bar{r})] \quad . \tag{10}$$

Notice that $A_z(\bar{r})$ in cylindrical coordinates depends on (R, ϕ, z) where R is the radius vector in these coordinates. If one fixes R and ϕ, then $A_z(\bar{r})$ will be a function of z alone along any line parallel to the z-axis. Equation (10) then becomes

$$\frac{\partial^2}{\partial z^2}[A_z(z) + k^2 A_z(z) = \frac{jk^2}{\omega} E_z^P(z) - \frac{\partial}{\partial z}[\frac{\partial}{\partial y} A_y(z)] \quad . \tag{1}$$

Equation (12) in the thesis is

$$\frac{d^2 f(x)}{dx^2} + k^2 f(x) = U(x) - \frac{\partial V(x)}{\partial x} \quad . \tag{12}$$

Examining equation (12) it is seen that equation (1) may be substituted directly into equation (12) and equation (17) which is the solution to equation (12) may be applied to equation (1). Thus it is seen that for equation (10) to use the solution given by equation (17) it must be in equation (1) form. This means equation (10) solution is good only along a line parallel to z-axis.

APPENDIX C: REDUCTION OF A DOUBLE INTEGRAL

In order to show the solution to equation (47) in general, the solution will be merely given and then differentiated to show that it does indeed give equation (47) even with arbitrary signs and the addition of a constant. The solution is

$$\int \left(\frac{jk}{R^2} + \frac{1}{R^3}\right)e^{-jkR}\cos k(\beta-\varepsilon)\,d\varepsilon = \frac{e^{-jkR}}{R[R-(\varepsilon-f)^2]}\left\{[\varepsilon-f]\cos(\beta-\varepsilon) - jR\sin k(\beta-\varepsilon)\right\}$$

where (1)

$$R = \left\{a^2 + g^2(\zeta) + [\varepsilon-f(\zeta)]^2\right\}^{1/2}.$$

Now let the solution in general be

$$\frac{e^{-jkR}}{R(R-[\varepsilon\pm f(n)\pm c]^2)}\left\{(\varepsilon\pm f\pm c)\cos k(\beta-\varepsilon) - jR\sin k(\beta-\varepsilon)\right\} \qquad (2)$$

and

$$R = \left\{a^2 + g^2(n) + [\varepsilon\pm f(n)\pm c]^2\right\}^{1/2}, \qquad (3)$$

where g and f are functions of the parameter n.

Now taking the derivative of the function

$$\frac{d}{d\varepsilon}\left\{\frac{e^{-jkR}}{R^3-R(\varepsilon\pm f\pm c)^2}[(\varepsilon\pm f\pm c)\cos k(\beta-\varepsilon) - jR\sin k(\beta-\varepsilon)]\right\} =$$

$$[(\varepsilon\pm f\pm c)\cos k(\beta-\varepsilon) - jR\sin k(\beta-\varepsilon)]\frac{d}{d\varepsilon}\left[\frac{e^{-jkR}}{R^3-R(\varepsilon\pm f\pm c)^2}\right] +$$

$$\left[\frac{e^{-jkR}}{R^3-R(\varepsilon\pm f\pm c)^2}\right]\frac{d}{d\varepsilon}[(\varepsilon\pm f\pm c)\cos k(\beta-\varepsilon) - jR\sin k(\beta-\varepsilon)]. \qquad (4)$$

Working on the first derivative in equation (4) gives

$$\frac{d}{d\varepsilon}\left[\frac{e^{-jkR}}{R^3-R(\varepsilon\pm f\pm c)^2}\right] = \left[\{R^3-R(\varepsilon\pm f\pm c)^2\}\left\{\frac{-jk(\varepsilon\pm f\pm c)e^{-jkR}}{R}\right\} -\right.$$

$$\left. e^{-jkR}\left\{\frac{3R^2(\varepsilon\pm f\pm c)}{R} - 2R(\varepsilon\pm f\pm c) - \frac{(\varepsilon\pm f\pm c)^2(\varepsilon\pm f\pm c)}{R}\right\}\right]\frac{1}{[R^3-R(\varepsilon\pm f\pm c)^2]^2} \; .$$

This reduces to give for the first part of equation (4)

$$\left[\frac{e^{-jkR}(\varepsilon\pm f\pm c)\left(\frac{-jk}{R} - \frac{1}{R^2}\right)}{R^3-R(\varepsilon\pm f\pm c)^2}\right]\left[(\varepsilon\pm f\pm c)\cos k(\beta-\varepsilon) - jR\sin k(\beta-\varepsilon)\right] \; . \qquad (5)$$

Similarly for the second derivative in equation (4) gives the following result

$$\left[(\varepsilon\pm f\pm c)k\sin k(\beta-\varepsilon) + \cos k(\beta-\varepsilon) + jkR\cos K(\beta-\varepsilon) - \frac{j\sin k(\beta-\varepsilon)(\varepsilon\pm f\pm c)}{R}\right]\times$$

$$\left[\frac{e^{-jkR}}{R^3-R(\varepsilon\pm f\pm c)^2}\right] \; . \qquad (6)$$

Adding equations (5) and (6) and factoring out the common elements results in

$$\frac{e^{-jkR}}{R^3-R(\varepsilon\pm f\pm c)^2}\left[(\varepsilon\pm f\pm c)k\sin k(\beta-\varepsilon) + \cos k(\beta-\varepsilon) + jkR\cos k(\beta-\varepsilon) - \right. \qquad (7)$$

$$\left.\frac{j(\varepsilon\pm f\pm c)\sin k(\beta-\varepsilon)}{R} + \frac{(\varepsilon\pm f\pm c)^2(-jk-\frac{1}{R})\cos k(\beta-\varepsilon)}{R} + \frac{(\varepsilon\pm f\pm c)(-jk-\frac{1}{R})(-jR\sin k[\beta-\varepsilon])}{R}\right].$$

Now start cancelling common terms

$$(\varepsilon\pm f\pm c)k\sin k(\beta-\varepsilon) - (\varepsilon\pm f\pm c)k\sin k(\beta-\varepsilon) = 0 \qquad (8a)$$

$$\frac{j(\varepsilon\pm f\pm c)\sin k(\beta-\varepsilon)}{R} - \frac{j(\varepsilon\pm f\pm c)\sin k(\beta-\varepsilon)}{R} = 0 \qquad (8b)$$

which gives as a result after some algebra

$$\frac{e^{-jkR}}{R^3-R(\varepsilon\pm f\pm c)^2}\left(\frac{\cos k(\beta-\varepsilon)}{R^2}\left\{[R^2-(\varepsilon\pm f\pm c)^2][1+jkR]\right\}\right) . \tag{9}$$

Cancelling common factor, one has

$$\frac{e^{-jkR}}{R}\left\{\frac{\cos k(\beta-\varepsilon)}{R^2}(1+jkR)\right\} = e^{-jkR}\cos(\beta-\varepsilon)\left[\frac{jk}{R^2}+\frac{1}{R^3}\right] \tag{10}$$

which is equation (47) in the thesis.

Thus equation (2) and (3) are required functions in general and equation (1) is the necessary function when there is no arbitrary constant or arbitrary signs in the integrand.

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Hallén type integral equations are formulated for an arbitrarily bent monopole above an infinite perfect ground plane and, by numerical solution techniques, current distributions are obtained. The coupled integral equations are founded upon the familiar electric scalar potential and magnetic vector potential. Numerical results are presented for several cases including different bent angles and are compared where possible with available data.

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